Philosophy 225– Symbolic Logic
Review Questions

1. Derive the last from the others where possible. Otherwise give an interpretation.

(a) $\forall x\, Fxx \vee \forall x \exists y Gxy$
$\forall x \forall y\, (Gxy \rightarrow Gyx)$
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$\exists x Gxx$

b) $\forall x \forall y\, (Fxy \rightarrow \exists y Fyx)$
$\forall x \exists y Fxy$
$\forall x\, [\exists y Fyx \rightarrow Fxx]$
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$\forall x Fxx$

c) $\forall x \exists y Fxy$
$\forall x \forall y\, (Fxy \rightarrow Gyy)$
____
$\forall x Gxx$

d) $\exists x \exists y\, (Fxy \wedge Gy)$
$\forall x\, (Hx \rightarrow \neg Gx)$
$\forall x\, (Hx \vee Fxx)$
____
$\exists x Fxx$

e) $\exists x \exists y\, (Fxy \wedge Gyy)$
$\forall x \forall y\, (Gxy \rightarrow \neg Fxx)$
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$\neg\, (\neg \forall x Fxx \wedge \neg \forall x Gxx)$

f) $\forall x \forall y \exists z (Fzx \wedge Fzy)$
$\forall x \forall y \forall z [(Fxy \wedge Fzy) \rightarrow Fxz]$
$\forall x \forall y \forall z [(Fxy \wedge Fab) \vee \neg Fyx]$
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$\forall x \forall y Fxy$ or $\forall x \exists y \neg Fxy$ [for each, either derive or show not a consequence]

2. Translate the following arguments into first-order logic. Then determine whether your translation of the conclusion is a consequence of your translations of the premises. If it is, give a derivation of the translation of the conclusion from the translation of the premises; if not, give an interpretation with a numerical domain in which the translations of the premises are true and in which the translation of the conclusion is false.

(a) Anybody who loves a model will not seek high political office.
Some models seek high political office.
Everybody who seeks high political office loves him/herself.
∴ Paris Hilton is a model.

(b) If Alice succeeds then somebody will enjoy success and congratulate herself.
Somebody will succeed.

Anybody who enjoys success feels good about herself.
∴ Somebody feels good about herself.

3. For each of [a] and [b], decide whether it is a consequence of the others. Prove your answers with an interpretation or a derivation.

(1) $\exists x \exists y Fxy$
$\forall x[\neg \forall z Fxz \to \forall y \neg Fyx]$

[a] $\forall x \forall y Fxy$ [b] $\forall x \exists y \neg Fxy$

(2) $\forall x \forall y\ (Fxy \lor Gyx)$
$\forall x\ (Fxx \to \forall y Gxy)$
$\forall x \forall y\ (Gxy \to \forall z Gyz)$

[a] $\forall x \forall y Gxy$ [b] $\forall x \exists y Fxy$

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