Additional Rules of Inference for Quantification Theory

UE (Universal Elimination): $\underline{\quad (\alpha)\Phi \quad}$. No strings attached.
$\qquad\qquad\qquad\qquad\qquad\quad \Phi\ \alpha/\beta$

UI (Universal Introduction): If you can derive $\Phi\ \alpha/\beta$, where β does not appear in any of the premises of $\Phi\ \alpha/\beta$, then you can generalize to (α)Φ. Be sure that you generalize on **all** occurrences of β; be sure that β does not appear in any of the premises of the line you generalize on.

EI (Existential Introduction): $\underline{\quad \Phi\ \alpha/\beta \quad}$. No strings attached.
$\qquad\qquad\qquad\qquad\qquad\quad (\exists\alpha)\Phi$

EE (Existential Elimination).

$$
\begin{array}{lll}
 & \cdot & \\
 & \cdot & \\
\Gamma & \text{(i)} \quad (\exists\alpha)\ \Phi & \\
 & \cdot & \\
 & \cdot & \\
\{j\} & \text{(j)} \quad \Phi\ \alpha/\beta & \text{Premise} \\
 & \cdot & \\
 & \cdot & \\
\Delta \cup \{j\} & \text{(k)} \quad \Psi & \\
\Gamma \cup \Delta & \text{(k+1)} \quad \Psi & \text{i, j, k EE}
\end{array}
$$

Restrictions: **β must not appear in line i, line k, or any premise of line k other than j**. Note that Ψ is simply rewritten. The function of the rule is to trade in premise j, the exemplar, for all the premises of (∃α) Φ.

Q (Quantifier): move a negation past a quantifier, and flip/flop the quantifier (universal turns to existential; existential turns to universal). Eliminate double negations if you wish. Rule Q can be used inside of WFFs.