

Phil 225- Symbolic Logic
HW 10 – possible answers

1a)	i	{1}	(1)	$(\exists x)(y)[Fxy \wedge (\exists z)Gzx]$	P
		{2}	(2)	$(x)(y)(z)\{(Gxy \wedge Fyz) \rightarrow Gxz\}$	P
		{3}	(3)	$(x)(y)(Gxy \rightarrow Gyx)$	P
	j	{4}	(4)	$(y)[(Fay \wedge (\exists z)Gza]$	P (for ES)
		{4}	(5)	$Fab \wedge (\exists z)Gza$	4 US
	I	{4}	(6)	$(\exists z)Gza$	5 T
	J	{7}	(7)	Gca	P (for ES)
		{4,7}	(8)	$Gca \wedge Fab$	5,7 T
		{2}	(9)	$(Gca \wedge Fab) \rightarrow Gcb$	2 US ³
		{2,4,7}	(10)	Gcb	8,9 T
		{3}	(11)	$Gcb \rightarrow Gbc$	3, US ²
		{2,3,4,7}	(12)	Gbc	10, 11 T
		{2,3,4,7}	(13)	$(y)Gyc$	12 UG
		K {2,3,4,7}	(14)	$(\exists x)(y)Gyx$	13 EG
	k	{2,3,4}	(15)	$(\exists x)(y)Gyx$	6,7,14 ES
		{1,2,3}	(16)	$(\exists x)(y)Gyx$	1,4,15 ES

1b) $(\exists x)(y)Fyx$ is NOT a consequence:

D: {0,1}

G: {<0,1>, <1,0>, <0,0>, <1,1>}

F: {<0,1>, <0,0>}

1c)	i	{1}	(1)	$(\exists x)(y)[Fxy \wedge (\exists z)Gzx]$	P
		{2}	(2)	$(x)(y)(z)\{(Gxy \wedge Fyz) \rightarrow Gxz\}$	P
		{3}	(3)	$(x)(y)(Gxy \rightarrow Gyz)$	P
		{4}	(4)	$(x)(y)(Fxy \rightarrow Fyx)$	P (for C)
	j	{5}	(5)	$(y)[(Fay \wedge (\exists z)Gza]$	P (for ES)
		{5}	(6)	$Fab \wedge (\exists z)Gza$	4 US
I		{5}	(7)	$(\exists z)Gza$	5 T
J		{8}	(8)	Gca	P (for ES)
		{5,8}	(9)	$Gca \wedge Fab$	6,8 T
		{2}	(10)	$(Gca \wedge Fab) \rightarrow Gcb$	2 US ³
		{2,5,8}	(11)	Gcb	9,10 T
		{3}	(12)	$Gcb \rightarrow Gbc$	3, US ²
		{2,3,5,8}	(13)	Gbc	11, 12 T
		{5}	(14)	$Fac \wedge (\exists z)Gza$	5 US
		{4}	(15)	$Fac \rightarrow Fca$	4 US ²
		{4,5}	(16)	Fca	14,15 T
		{2}	(17)	$(Gbc \wedge Fca) \rightarrow Gba$	2 US ³
		{2,3,4,5,8}	(18)	Gba	13,16,17 T
		{5}	(19)	$Fad \wedge (\exists z)Gza$	5 US
		{2,3,4,5,8}	(20)	$Gba \wedge Fad$	18,19 T
		{2}	(21)	$(Gba \wedge Fad) \rightarrow Gbd$	2 US ³
K		{2,3,4,5,8}	(22)	Gbd	20,21 T
k		{2,3,4,5}	(23)	Gbd	7,8,22 ES
		{1,2,3,4}	(24)	Gbd	1,5,23 ES
		{1,2,3,4}	(25)	$(y)Gby$	24 UG
		{1,2,3,4}	(26)	$(x)(y)Gxy$	25 UG
		{1,2,3}	(27)	$(x)(y)(Fxy \rightarrow Fyx) \rightarrow (x)(y)Gxy$	4,26 C

2)	(a) Valid				
		{1}	(1)	Fa	P (for C)
		{1}	(2)	$Ga \vee Fa$	1 T
		{1}	(3)	$(\exists y)(Gy \vee Fy)$	2 EI
		\wedge	(4)	$Fa \rightarrow (\exists y)(Gy \vee Fy)$	1,3 C
		\wedge	(5)	$(x)[Fx \rightarrow (\exists y)(Gy \vee Fy)]$	4 UI

	(b) Inconsistent				
	i	{1}	(1)	$(\exists x)(y)(Fxy \leftrightarrow \neg Fyy)$	P (for RAA)
	j	{2}	(2)	$(y)(Fay \leftrightarrow \neg Fyy)$	P (for EE)
		{2}	(3)	$(Faa \leftrightarrow \neg Faa)$	2 UE
k		{2}	(4)	$P \wedge \neg P$	3 T
		{1}	(5)	$P \wedge \neg P$	1,2,4 EE
		\wedge	(6)	$\neg(\exists x)(y)(Fxy \leftrightarrow \neg Fyy)$	1 5 RAA

(c) Neither

Here is an interpretation in which it is true:

$D = \{0\}$ $F = \{0\}$ $G = \{0\}$

Here is one in which it is false:

$D = \{0\}$ $F = \{0\}$ $G = \Lambda$