

Phil 225 -- Symbolic Logic
HW 9 Answers

(1) (b) is a consequence

	{1}	(1)	$(x)(y)(Fxy \rightarrow (\exists z)Gxz)$	P
i	{2}	(2)	$(\exists x)(\exists y)Fxy$	P
	{3}	(3)	$(x)(y)(Fxy \rightarrow (z)Fxz)$	P
$(x)(\exists y)Gxy$	{4}	(4)	$(x)(y)(Fxy \rightarrow (\exists z)Fyz)$	P
I j	{5}	(5)	$(\exists y)Fay$	P (for EE)
J	{6}	(6)	Fab	P (for EE)
	{3}	(7)	$Fab \rightarrow (z)Faz$	3 UE ²
	{3,6}	(8)	$(z)Faz$	6,7 T
	{3,6}	(9)	Fac	8 UE
	{4}	(10)	$Fac \rightarrow (\exists z)Fcz$	4 UE ²
I'	{3,4,6}	(11)	$(\exists z)Fcz$	9,10 T
J'	{12}	(12)	Fcd	P (for EE)
	{1}	(13)	$Fcd \rightarrow (\exists z)Gcz$	1 UE ²
K'	{1,12}	(14)	$(\exists z)Gcz$	12,13 T
I''	{1,3,4,6}	(15)	$(\exists z)Gcz$	11,12,14 EE
J''	{16}	(16)	Gce	P (for EE)
K''	{16}	(17)	$(\exists y)Gcy$	16 EI
	{1,3,4,6}	(18)	$(\exists y)Gcy$	15,16,17 EE
K	{1,3,4,6}	(19)	$(x)(\exists y)Gxy$	18 UI
k	{1,3,4,5}	(20)	$(x)(\exists y)Gxy$	5,6,19 EE
	{1,2,3,4}	(21)	$(x)(\exists y)Gxy$	2,5,20 EE

To show that (a) is **not** a consequence, let $D = \{0,1\}$

F: $\{<0,0>, <0,1>, <1,0>, <1,1>\}$

G: $\{<0,0>, <1,0>\}$

	{1}	(1)	$P \rightarrow (Q \vee R)$	P
	{2}	(2)	$\neg Q \rightarrow P$	P
R	{3}	(3)	$Q \rightarrow (R \wedge S)$	P
	\wedge	(4)	$Q \vee \neg Q$	TH
	{5}	(5)	Q	P (for C)
	{3,5}	(6)	$R \wedge S$	5,3 MP
	{3,5}	(7)	R	6 CE
	{3}	(8)	$Q \rightarrow R$	5,7 C
	{9}	(9)	$\neg Q$	P (for C)
	{2,9}	(10)	P	2,9 MP
	{1,2,9}	(11)	$Q \vee R$	10,1 MP
	{1,2,9}	(12)	R	9,11 DE
	{1,2}	(13)	$\neg Q \rightarrow R$	9,12 C
	{1,2,3}	(14)	R	4,8,13 DE

3)	i	{1}	(1)	$(\exists x)(\exists y)(Fxy \wedge Gya)$	P
		{2}	(2)	$(x)[(\exists y)Fxy \rightarrow (z)Gxz]$	P
		{3}	(3)	$(x)(y)(z)[Gxy \rightarrow Gyz]$	P
	I j	{4}	(4)	$(\exists y)(Fby \wedge Gya)$	P (for ES) (**Use 'b')
	J	{5}	(5)	$Fbc \wedge Gca$	P (for ES) not 'a';
		{2}	(6)	$(\exists y)Fby \rightarrow (z)Gbz$	2, US otherwise
		{5}	(7)	Fbc	5, T ES won't
		{5}	(8)	$(\exists y)Fby$	7, EG (work.)
		{2,5}	(9)	$(z)Gbz$	8,6 T
		{2,5}	(10)	Gbd	9, US
		{3}	(11)	$Gbd \rightarrow Gde$	3, US ³
	K	{2,3,5}	(12)	Gde	10, 11 T
	k	{2,3,4}	(13)	Gde	4,5,12, ES
		{1,2,3}	(14)	Gde	1,4,13, ES
		{1,2,3}	(15)	$(y)Gdy$	14, UG
		{1,2,3}	(16)	$(x)(y)Gxy$	15, UG

4a)		{1}	(1)	$(x)Fx \wedge (\exists y)(Gy \vee Hy)$	P (for C)
		{2}	(2)	$(x)\neg Gx$	P (for C)
		{1}	(3)	$(x)Fx$	1 T
		{1}	(4)	Fa	3 US
	i	{1}	(5)	$(\exists y)(Gy \vee Hy)$	1 T
	j	{6}	(6)	$Ga \vee Ha$	P (for ES)
		{2}	(7)	$\neg Ga$	2 US
		{1,2,6}	(8)	$Fa \wedge Ha$	4,6,7 T
	k	{1,2,6}	(9)	$(\exists y)(Fy \wedge Hy)$	8 EG
		{1,2}	(10)	$(\exists y)(Fy \wedge Hy)$	5,6,9 ES
		{1}	(11)	$(x)\neg Gx \rightarrow (\exists y)(Fy \wedge Hy)$	2,10 C
	Λ		(12)	$[(x)Fx \wedge (\exists y)(Gy \vee Hy)] \rightarrow [(x)\neg Gx \rightarrow (\exists y)(Fy \wedge Hy)]$	1,10C

4b) $(\exists x)[(\exists y)Fxy \wedge (x)(y)(Fxy \rightarrow Gyy)] \rightarrow (x)(\exists y)Gxy$

Not Valid: D = {0,1}
 F: {<0,0>}
 G: {<0,0>}

4c)		{1}	(1)	$\neg(\exists x)(Fx \rightarrow (x)Fx)$	P (for RAA)
		{1}	(2)	$(x)\neg(Fx \rightarrow (x)Fx)$	1 Q
		{1}	(3)	$\neg(Fa \rightarrow (x)Fx)$	2 US
		{1}	(4)	$Fa \wedge \neg(x)Fx$	3 T
		{1}	(5)	Fa	4 T
		{1}	(6)	$(x)Fx$	5 UG
		{1}	(7)	$(x)Fx \wedge \neg(x)Fx$	4,6 T
	Λ		(8)	$(\exists x)(Fx \rightarrow (x)Fx)$	1,7 RAA