

Philosophy 225 -- Symbolic Logic  
HW 8 Possible Answers

1a)		{1}	(1)	$(x)(\exists y)Fxy$	P
		{2}	(2)	$(x)(y)(Fxy \rightarrow Gyy)$	P
		{3}	(3)	$(x)(y)(Fxy \rightarrow Fyx)$	P
	i	{1}	(4)	$(\exists y)Fay$	1, UE
	j	{5}	(5)	$Fab$	P (for EE)
		{3}	(6)	$Fab \rightarrow Fba$	3, UE <sup>2</sup>
		{2}	(7)	$Fba \rightarrow Gaa$	2, UE <sup>2</sup>
	k	{2,3,5}	(8)	$Gaa$	5,6,7, T
		{1,2,3}	(9)	$Gaa$	4,5,8, EE
		{1,2,3}	(10)	$(x)Gxx$	9, UI

\*\* Note that you must do EE before doing UI.

1b)	i	{1}	(1)	$(\exists x)(y)Fxy$	P
	j	{2}	(2)	$(y)Fay$	P (for EE)
		{2}	(3)	$Fab$	2 UE
		{2}	(4)	$(\exists x)Fxb$	3 EI
	k	{2}	(5)	$(y)(\exists x)Fxy$	4 UI
		{1}	(6)	$(y)(\exists x)Fxy$	1,2,5 EE

2a) IS a consequence:

		{1}	(1)	$(x)(y)(Fxy \rightarrow Gy)$	P
		{2}	(2)	$(x)(Gx \rightarrow \neg Hxx)$	P
	i	{3}	(3)	$(\exists x)Hxx$	P
	j	{4}	(4)	$Haa$	P (for EE)
		{2}	(5)	$Ga \rightarrow \neg Haa$	2, UE
		{1}	(6)	$Fba \rightarrow Ga$	1, UE <sup>2</sup>
		{1,2,4}	(7)	$\neg Fba$	4,5,6, T
		{1,2,4}	(8)	$(y)\neg Fya$	7, UI
	k	{1,2,4}	(9)	$(\exists x)(y)\neg Fyx$	8, EI
		{1,2,3}	(10)	$(\exists x)(y)\neg Fyx$	3,4,9 EE

2b) is NOT a consequence:

D = {0}  
F:  $\Lambda$   
G:  $\Lambda$   
H: {<0,0>}

3)		{1}	(1)	$(y)(\exists x)Fxy$	P
	i	{1}	(2)	$(\exists x)Fxa$	1 UE
	j	{3}	(3)	$Fba$	P (for EE)

Now we have to use UI to generalize on 'a' in line 3. That is, we would like to do this:

		{3}	(4)	$(y)Fby$	3 UI
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But the constant we are generalizing on, namely 'a', appears in the premise of line (3), namely premise {3}. Hence we cannot use UI. In this case there is no way around the problem: by soundness, the conclusion cannot be derived from the premise, since it is not a consequence of it. We see again that our rules are safe (i.e., sound) since they block an inference that would lead us to a bad conclusion.

4a)		{1}	(1)	$P \rightarrow -(S \wedge -T)$	P
		{2}	(2)	$-P \rightarrow -S$	P
	-S	{3}	(3)	$-T$	P
		{4}	(4)	$S$	P (for RAA)
		{2,4}	(5)	$P$	2,4 TH (Contra)
		{1,2,4}	(6)	$-(S \wedge -T)$	1,5 MP
		{1,2,4}	(7)	$-S \vee -- -T$	6 R (Contra)
		{1,2,3,4}	(8)	$-S$	3,7 DE
		{1,2,3,4}	(9)	$S \wedge -S$	4,8 CE
		{1,2,3}	(10)	$-S$	4,9 RAA

4b)		{1}	(1)	$-[R \vee (-S \rightarrow -T)]$	P
	-Q	{2}	(2)	$-[(R \rightarrow S) \rightarrow -(-R \rightarrow -Q)]$	P
		{1}	(3)	$-R \wedge -(-S \rightarrow -T)$	1 R (DeM)
		{2}	(4)	$(R \rightarrow S) \wedge -- (-R \rightarrow -Q)$	2 R (DeM)
		{2}	(5)	$-- (-R \rightarrow -Q)$	4 CE
		{2}	(6)	$-R \rightarrow -Q$	5 R (DN)
		{1}	(7)	$-R$	3 CE
		{1,2}	(8)	$-Q$	6,7 MP