

Philosophy 225 -- Symbolic Logic  
HW 7 Possible Answers

(a)	{1}	(1) $P \leftrightarrow (Q \rightarrow R)$	P
	R {2}	(2) $\neg (Q \vee P)$	P
	{2}	(3) $\neg Q \wedge \neg P$	2 R (De Morgan)
	{2}	(4) $\neg P$	3 CE
	{1,2}	(5) $\neg (Q \rightarrow R)$	1,4 BE
	{1,2}	(6) $Q \wedge \neg R$	5 R (De Morgan)
	{2}	(7) $\neg Q$	3 CE
	{1,2}	(8) Q	6 CE
	{1,2}	(9) $Q \wedge \neg Q$	7,8 CI
	{1,2}	(10) R	9 RAA

So we see that the premises were actually inconsistent. Hence we could have derived any WFF from them.

(b)	{1}	(1) $\neg P \rightarrow (Q \vee R)$	P
	{2}	(2) $(S \vee Q) \rightarrow \neg R$	P
	P {3}	(3) $\neg (S \rightarrow Q)$	P
	{3}	(4) $S \wedge \neg Q$	3 R (De Morgan)
	{3}	(5) S	4, CE
	{3}	(6) $\neg Q$	4, CE
	{3}	(7) $S \vee Q$	5, DI
	{2,3}	(8) $\neg R$	2,7 MP
	X {9}	(9) $\neg P$	P (for RAA)
	{1,9}	(10) $Q \vee R$	9,1 MP
	{1,2,3,9}	(11) Q	8,10 DE <sub>1</sub>
	{1,2,3,9}	(12) $Q \wedge \neg Q$	6,11 CI
	{1,2,3}	(13) P	9,12 RAA

After (8), there seems to be no easy way to go. I could use lines 6 and 8 to try to derive the negation of the consequent of line 1, but that's a little messy, and I notice that the opposite of what I'm trying to derive, namely  $\neg P$ , lets me make quick progress using line 1 and MP; so I decide instead to declare myself stuck.

(c)	{1}	(1) $(Q \rightarrow R) \rightarrow \neg P$	P
	{2}	(2) $S \vee R$	P
	{3}	(3) $(U \rightarrow \neg P) \rightarrow \neg Q$	P
$\neg T \vee S$	{4}	(4) $T \rightarrow Q$	P
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S	{5}	(5) T	P (for C)
	{4,5}	(6) Q	4,5 MP
	{3,4,5}	(7) $\neg(U \rightarrow \neg P)$	3,6 TH (Contra)
	{3,4,5}	(8) $U \wedge \neg\neg P$	7 R (Contra)
	{3,4,5}	(9) $\neg\neg P$	8 CE
	{3,4,5}	(10) P	9 R (Neg)
	{1,3,4,5}	(11) $\neg(Q \rightarrow R)$	1,11 TH (De Morgan)
	{1,3,4,5}	(12) $Q \wedge \neg R$	11 R (De Morgan)
	{1,3,4,5}	(13) $\neg R$	12 CE
	{1,2,3,4,5}	(14) S	2, 13 DE <sub>1</sub>
	{1,2,3,4}	(15) $T \rightarrow S$	5,14 C
	{1,2,3,4}	(16) $\neg\neg T \rightarrow S$	15 R (Neg)
	{1,2,3,4}	(17) $\neg T \vee S$	16 D

At (5) I am trying to derive the disjunction  $\neg T \vee S$ . I can instead trade this in for its corresponding conditional,  $T \rightarrow S$ . Accordingly, I take T as a premise (for rule C), hoping to derive S. At the end, I will trade the conditional  $T \rightarrow S$  back in for the disjunction  $\neg T \vee S$ .

(d)	{1}	(1) $P \rightarrow (\neg Q \wedge R)$	P
	{2}	(2) $S \rightarrow (Q \vee \neg R)$	P
	$\neg Q \rightarrow \neg S$ {3}	(3) $\neg P \rightarrow R$	
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Q	{4}	(4) S	P (for C)
	{2,4}	(5) $Q \vee \neg R$	2,4 MP
X	{6}	(6) $\neg Q$	P (for RAA)
	{2,4,6}	(7) $\neg R$	5,6 DE <sub>1</sub>
	{2,3,4,6}	(8) P	7,3 TH (Contra)
	{1,2,3,4,6}	(9) $\neg Q \wedge R$	1,8 MP
	{1,2,3,4,6}	(10) R	9 CE
	{1,2,3,4,6}	(11) $R \wedge \neg R$	7,10 CI
	{1,2,3,4}	(12) Q	6,11 RAA
	{1,2,3}	(13) $S \rightarrow Q$	4,12 C
	{1,2,3}	(14) $\neg Q \rightarrow \neg S$	13 R (or TH) (Contra)

Note: at (4) I took S as a premise because it looked easier to derive the contrapositive ( $S \rightarrow Q$ ) than the original ( $\neg Q \rightarrow \neg S$ ). This is because it looked as though the premise S would yield more information than the premise  $\neg Q$ . Then at (6) I didn't see anywhere easy to go, so I declared myself stuck and took  $\neg Q$ , the negation of what I was trying to derive (Q), as a premise for RAA.2b)

## 2) Ch. 7, Problem 1

1b)	{1}	(1) $(x)(y)Fxy$	P
	{1}	(2) $(y)Fay1$ , UE	
	{1}	(3) Fab	2, UE
	{1}	(4) $(y)Fyb$	3, UI
	{1}	(5) $(x)Fyx$	4, UI
1e)	{1}	(1) $(x)(Fx \wedge Gx)$	P
	{1}	(2) Fa $\wedge$ Ga	1, UE
	{1}	(3) Fa	2, T
	{1}	(4) $(x)Fx$	3, UI
	{1}	(5) Ga	4, T
	{1}	(6) $(x)Gx$	5, UI
	{1}	(7) $(x)Fx \wedge (x)Gx$	4,6 T

1f)		{1}	(1)	$(x)Fx \vee (x)Gx$	P
	X	{2}	(2)	$\neg(x)(Fx \vee Gx)$	P (for RAA)
	i	{2}	(3)	$(\exists x) \neg(Fx \vee Gx)$	2, Q
	j	{4}	(4)	$\neg(\underline{Fa} \vee Ga)$	P (for EE)
		{4}	(5)	$\neg Fa$	4, T
		{4}	(6)	$\neg Ga$	4, T
		{4}	(7)	$(\exists x) \neg Fx$	5, EI
		{4}	(8)	$(\exists x) \neg Gx$	6, EI
		{4}	(9)	$\neg(x)Fx$	7, Q
		{4}	(10)	$\neg(x)Gx$	8, Q
	k	{4}	(11)	$P \wedge \neg P$	1,9,10 T
		{1,2}	(12)	$P \wedge \neg P$	3,4,11 EE
		{1}	(13)	$(x)(Fx \vee Gx)$	2, 12 RAA
1g)		{1}	(1)	$(x)(Fx \rightarrow (Gx \vee Hx))$	P
		{2}	(2)	$Ga \leftrightarrow (Ha \wedge \neg Ga)$	P
		{1}	(3)	$Fa \rightarrow (Ga \vee Ha)$	1, UE
		{1,2}	(4)	$\neg Fa$	2,3 T
1h)		{1}	(1)	$(x)(y)(z)[(Fxy \wedge Fyz) \rightarrow Fxz]$	P
		{2}	(2)	$(x)\neg Fxx$	P
		{1}	(3)	$(y)(z)[(Fay \wedge Fyz) \rightarrow Faz]$	1, UE
		{1}	(4)	$(z)[(Fab \wedge Fbz) \rightarrow Faz]$	3, UE
		{1}	(5)	$(Fab \wedge Fba) \rightarrow Faa$	4, UE
		{2}	(6)	$\neg Faa$	2, UE
		{1,2}	(7)	$Fab \rightarrow \neg Fba$	5,6, T
		{1,2}	(8)	$(y)(Fay \rightarrow \neg Fya)$	7, UI
		{1,2}	(9)	$(x)(y)(Fxy \rightarrow \neg Fyx)$	8, UI
1j)		{1}	(1)	$(x)(Fx \vee Gx)$	P
	i	{2}	(2)	$(\exists x) \neg Gx$	P
		{3}	(3)	$(x)(Hx \rightarrow \neg Fx)$	P
	j	{4}	(4)	$\neg \underline{Ga}$	P (for EE)
		{1}	(5)	$Fa \vee Ga$	1, UE
		{1,4}	(6)	$Fa$	4,5, T
		{3}	(7)	$Ha \rightarrow \neg Fa$	3, UE
		{1,3,4}	(8)	$\neg Ha$	6,7 T
	k	{1,3,4}	(9)	$(\exists x) \neg Hx$	8, EI
		{1,2,3}	(10)	$(\exists x) \neg Hx$	2,4,9, EE