Phil 225 -- Symbolic Logic
Possible Answers to HW 3
Spring 2012

1) (a) D = {1,2}
G: {1}
F: {<1,2>, <2,1>}

(b) D = {0,1}
P: {0}
Q: Λ
R: Λ

(c) D = **N**
F: {<x,y> | x<y}

2) (a) There must be at least two elements. Since $|=^{I}$ Fa but $|\neq^{I}$ Fb, I(a) ≠ I(b). This is possible so long as there are two elements in the domain.

(b) (Fab ∧ -Fcb ∧ -Fac ∧ -Fba) or
(Fabc ∧ -Faac ∧ -Fabb ∧ -Faba) or
(Fa ∧ -Fb ∧ Gb ∧ -Gc ∧ Ha ∧ -Hc). There are lots of other ways.

(c) The conjunction of the three sentences of 11(c) is satisfiable only in infinite domains. To see this, pick any x (call it a). The first sentence requires a to bear F to something, say b. The 3rd sentence requires that b be different from a, so b is a new object. But now by the first sentence b must be related to something, c. By the same reasoning, c must be different from b. In addition, if c were just a, then Fab and Fbc, so by the second sentence Fca; but if c were the same as a, that would contradict the third sentence. Hence c must be different from a. So c is new. But now c must bear F to something. And so on… [This will contrast nicely with its converse. It will turn out that there are no sentences satisfiable only in finite domains.]

3) a) Not a consequence. D = {1,2} Second disjunct of sent. 1
F: Λ is true. Sent. 2 is true
G: {<1,2>,<2,1>} vacuously.

In this one the trick is to make the first sentence true by making its second disjunct true. If the first disjunct becomes true, the game is over, because then the second sentence will force the third to be true (and then some).

b) (x)Fxx is a consequence.

c) Not a consequence. D = {0,1}
F: {<0,1>,<1,1>}
G: {<1,1>}

4) D = {0,1}
F: {<0,0>,<1,1>}
G: {<0,0>,<0,1>,<1,0>,<1,1>}

sentence 1: Pick y as the same as x
sentence 2: All pairs are in G, so the consequent is true for all choices
sentence 3: same as 2
sentence 4: pick z the same as x, and y as different from x

This is a pretty cheap way to make all of them true. The problem is a hard one because it's hard to figure out how to understand the 4th sentence. But by the same token it is easy because once you have made the 1st and 4th sentences true, you can just add stuff into G profligately without having to go back and make changes in F. I could have (and at some point undoubtedly will) give a much more diabolical problem in which there are sentences as nasty as the 4th one here, but where you have to keep coming back to them as other sentences interact with what you have already done.

5) (a) If –A is valid then –A is true in every interpretation, and so (by clause 3 of the truth definition) A is false in every interpretation. Hence there is no interpretation in which A is true (by def. of 'false'), and so *a fortiori*, there is no interpretation in which A is true *and* B is false. Hence (by def. of 'consequence'), B is a consequence of A. But this holds for any sentence B.

(b) Spose {A, B} is inconsistent, and let C be an arbitrary sentence. There is no interpretation in which both A and B are true. *A fortiori* there is no interpretation in which A and B are both true and C is false. So (by def. of 'consequence') C is a consequence of {A, B}.

(c) Spose A is valid and let Γ be any set of sentences. Since A is valid, it is true in every interpretation (by def. of 'valid') and so there is no interpretation in which it is false. *A fortiori*, there is no interpretation in which all the sentences of Γ are true *and* A is false; so (by def. of 'consequence') A is a consequence of Γ. But Γ was arbitrary; so A is a consequence of every set of sentences.