

Phil 225 – Symbolic Logic
HW 9 Possible Answers
April 18, 2011

15.15:

- 1) {2,4}
- 2) {2,4}
- 3) {2,3,4,5,6,8}
- 4) \emptyset
- 5) {2,3,4,5,6,7,8,9}
- 6) {2,3,4,5,7,9}
- 7) {2,3,4,5}

15.17 See attached Fitch proof

15.23:

a has 2 elements; b has 1 element. So $a \neq b$

15.40-1

a) Suppose R is symmetric, and suppose $\langle x,y \rangle \in R$. By symmetry, $\langle y,x \rangle \in R$, so by def of R^{-1} , $\langle x,y \rangle \in R^{-1}$. b) Conversely, suppose $\langle x,y \rangle \in R^{-1}$. Then by def of inverse, $\langle y,x \rangle \in R$. So since R is symmetric, $\langle x,y \rangle \in R$. Hence $\langle x,y \rangle \in R$ iff $\langle x,y \rangle \in R^{-1}$, so by extensionality, $R = R^{-1}$.

15.40 – 2:

a) Suppose $\langle x,y \rangle \in (R^{-1})^{-1}$. Then by def of inverse, $\langle y,x \rangle \in R^{-1}$, so by def of inverse again, $\langle x,y \rangle \in R$. b) Conversely, by the same reasoning, if $\langle x,y \rangle \in R$, then $\langle x,y \rangle \in (R^{-1})^{-1}$. Hence $\langle x,y \rangle \in R$ iff $\langle x,y \rangle \in (R^{-1})^{-1}$, so by extensionality, $R = (R^{-1})^{-1}$.

We could also do these more simply as chains of bi-conditionals. Thus: $\langle x,y \rangle \in R$ iff $\langle y,x \rangle \in R^{-1}$ (by def of inverse) iff $\langle x,y \rangle \in (R^{-1})^{-1}$ (again by def of inverse). Where the argument forward and backward is the same, this is a much simpler way of establishing a bi-conditional. But often the arguments are different, so it must be spelled out. The first one would also be simpler by this method.

15.54

$\{\emptyset, \{2\}, \{3\}, \{4\}, \{2,3\}, \{2,4\}, \{3,4\}, \{2,3,4\}\}$

15.56

$\{\emptyset, \{2\}\}$

15.58

$\{\emptyset, \{\emptyset\}, \{\{2\}\}, \{\{3\}\}, \{\{2,3\}\}, \{\emptyset, \{2\}\}, \{\emptyset, \{3\}\}, \{\emptyset, \{2,3\}\}, \{\{2\}, \{3\}\}, \{\{2\}, \{2,3\}\}, \{\{3\}, \{2,3\}\}, \{\{2\}, \{3\}, \{2,3\}\}, \{\emptyset, \{3\}, \{2,3\}\}, \{\emptyset, \{2\}, \{2,3\}\}, \{\emptyset, \{2\}, \{3\}\}, \{\emptyset, \{2\}, \{3\}, \{2,3\}\}\}$