Introduction to Logic
Summary

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Multiple Logics

Propositional Logic (logical operators)

$$(p \Rightarrow q) \Leftrightarrow (\neg p \vee q)$$

Relational Logic (variables and quantifiers)

$$(\forall x.p(x)) \vee (\exists x.q(x,x))$$

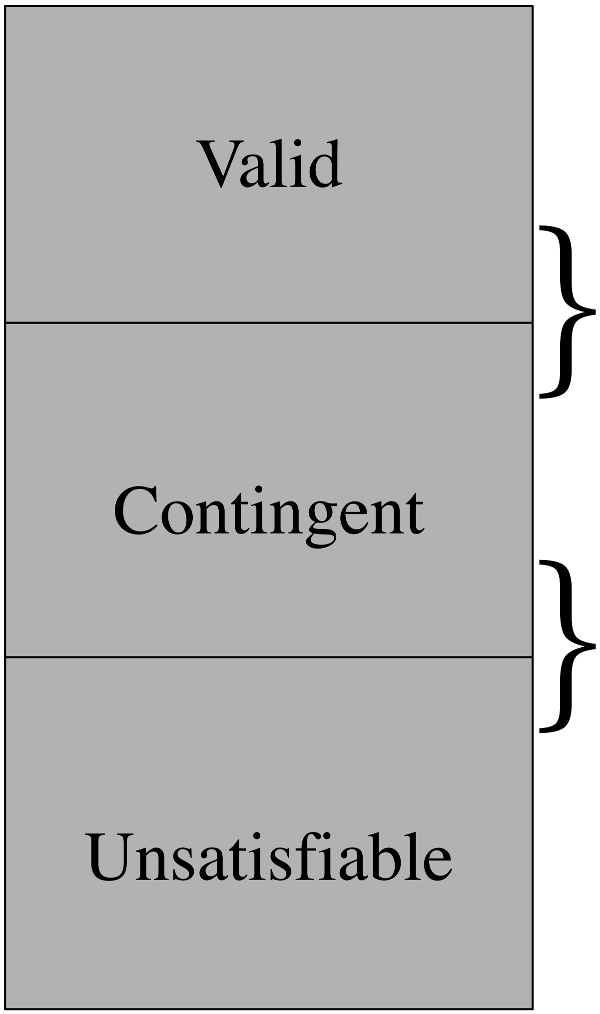
Functional Logic (compound terms)

$$\forall x.(p(x) \Leftrightarrow p(f(x)) \wedge p(g(x)))$$

Properties of Sentences

Valid	A sentence is *valid* if and only if *every* interpretation satisfies it.
Contingent	A sentence is *contingent* if and only if *some* interpretation satisfies it and *some* interpretation falsifies it.
Unsatisfiable	A sentence is *unsatisfiable* if and only if *no* interpretation satisfies it.

Properties of Sentences



A sentence is *satisfiable* if and only if it is either valid or contingent.

A sentence is *falsifiable* if and only if it is contingent or unsatisfiable.

Relationships

A sentence ϕ is *logically equivalent* to a sentence ψ if and only if every truth assignment that satisfies ϕ satisfies ψ *and* every truth assignment that satisfies ψ satisfies ϕ.

A sentence ϕ is *consistent with* a sentence ψ if and only if there is a truth assignment that satisfies both ϕ and ψ.

A set of premises Δ *logically entails* a conclusion φ (written as $\Delta \models \varphi$) if and only if every interpretation that satisfies the premises also satisfies the conclusion.

Metatheorems

Propositional Metatheorems:

- Equivalence Theorem
- Deduction Theorem
- Unsatisfiability Theorem
- Consistency Theorem
- Substitution Theorem

These theorems also hold in Relational Logic
*but only for **closed** sentences (no free variables).*

Determining Logical Entailment

$$\{p(a) \vee p(b), \forall x.(p(x) \Rightarrow q(x))\} \models \exists x.q(x)?$$

$p(a)$	$p(b)$	$q(a)$	$q(b)$	$p(a) \vee p(b)$	$\forall x.(p(x) \Rightarrow q(x))$	$\exists x.q(x)$
1	1	1	1	1	1	1
1	1	1	0	1	0	1
1	1	0	1	1	0	1
1	1	0	0	1	0	0
1	0	1	1	1	1	1
1	0	1	0	1	1	1
1	0	0	1	1	0	1
1	0	0	0	1	0	0
0	1	1	1	1	1	1
0	1	1	0	1	0	1
0	1	0	1	1	1	1
0	1	0	0	1	0	0
0	0	1	1	0	1	1
0	0	1	0	0	1	1
0	0	0	1	0	1	1
0	0	0	0	0	1	0

Proof

$$\{\forall x.(p(x) \Rightarrow q(x)), \forall x.(q(x) \Rightarrow r(x))\} \models \forall x.(p(x) \Rightarrow r(x))$$

1.	$\forall x.(p(x) \Rightarrow q(x))$	Premise
2.	$\forall x.(q(x) \Rightarrow r(x))$	Premise
3.	$p([c]) \Rightarrow q([c])$	UE: 1
4.	$q([c]) \Rightarrow r([c])$	UE: 2
5.	$\mid p([c])$	Assumption
6.	$\mid q([c])$	IE: 5, 3
7.	$\mid r([c])$	IE: 6, 4
8.	$p([c]) \Rightarrow r([c])$	II: 5, 7
9.	$\forall x.(p(x) \Rightarrow r(x))$	UI: 8

Soundness and Completeness

A proof system is *sound* if and only if every provable conclusion is logically entailed.

$$\text{If } \Delta \vdash \phi \text{, then } \Delta \models \phi.$$

A proof system is *complete* if and only if every logical conclusion is provable.

$$\text{If } \Delta \models \phi \text{, then } \Delta \vdash \phi.$$

Soundness and Completeness

Fitch is sound and complete for Propositional Logic.

$\Delta \vdash \phi$ if and only if $\Delta \models \phi$.

Fitch is sound and complete for Relational Logic.

$\Delta \vdash \phi$ if and only if $\Delta \models \phi$.

Fitch (w/ induction) is sound for Functional Logic, not complete.

If $\Delta \vdash \phi$, then $\Delta \models \phi$.

What's Next?

Automated Theorem Proving

Building computers that can prove theorems

CS 257 - Automate Reasoning

Logic Programming

Using Logic to represent knowledge, program computers

CS 151 - Logic Programming

CS 227B - General Game Playing

Advanced Logics

Metadata, Paraconsistent Reasoning, Probabilities

Phil 161

What's Next?

Final Exam