Introduction to Logic
Introduction

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Hello. And welcome to this introductory course on Logic. My name is Michael Genesereth. I am a professor of Computer Science at Stanford University, and I have been teaching this course for more than 30 years. In these videos, I will be talking you through the material of the course.

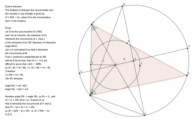
Hermione Granger

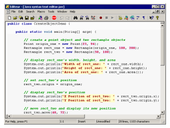
This isn't magic - it's ***logic*** *- a puzzle. A lot of the greatest wizards haven't got an ounce of logic; they'd be stuck here forever."*

- Harry Potter and The Sorcerer's Stone

Hermione Granger got it right when, facing the potion-master's test in Harry Potter, she said: "This isn't magic - it's *logic* - a puzzle. A lot of the greatest wizards haven't got an ounce of logic; they'd be stuck here forever." In the real world, we are better off. We use Logic in just about everything we do.

Uses of Logic





We use it in our professional lives - in proving mathematical theorems, in debugging computer programs, in medical diagnosis, and in legal reasoning.

Uses of Logic

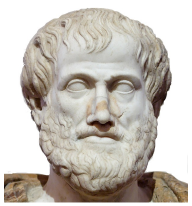
*"Whether I am on a **soccer field** or at a **robotics competition**, I face a lot of situations where logic is necessary to make decisions."*

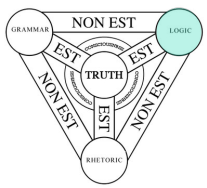
*"I have always loved **puzzles** and like to solve challenging problems."*

*"**Math** classes aren't the only classes that require logic; in AP **History** I am often called upon to recognize patterns and cycles spanning centuries, while in **English** classes I need to write persuasive essays."*

And we use Logic in our personal lives - in playing games, solving puzzles, and doing school assignments, not just in math, but also history and English and other subjects.

Greek Trivium





The ancient Greeks thought Logic sufficiently important that it was one of the three subjects in the Greek educational Trivium, along with Grammar and Rhetoric.

Current Curriculum

✓ Biology	✓ Algebra
✓ Chemistry	✓ Geometry
✓ Physics	✓ Calculus
✓ Computer Science	× Logic

Calculus is the mathematics of Physics.
Logic is the mathematics of Computer Science.

Oddly, Logic occupies a relatively small place in the modern school curriculum. We have courses in the sciences and various branches of mathematics, but very few secondary schools offer courses in Logic. Given the importance of the subject, this is surprising.

Calculus is important to physics. And it is widely taught at the high school level. Logic is important in all of these disciplines, and it is *essential* in computer science. Yet it is rarely offered as a standalone course, making it more difficult for students to succeed at the college level and get better quality jobs.

This Course

Target Audience
University students
Talented High School Students
Interested professionals

Prerequisites
Sets and Set Operations (union, intersection, complement)
Symbolic Manipulation (e.g. high school algebra)

This course is the sort of introduction to Logic we think should be available to fill this need. It is intended primarily for university students. However, it has been used by motivated secondary school students and post-graduate professionals interested in honing their logical reasoning skills.

There are just two prerequisites. The course presumes that the student understands sets and set operations, such as union, intersection, and complement. The course also presumes that the student is comfortable with symbolic mathematics, at the level of high-school algebra. If you have those prerequisites, you should be good to go.

Elements of Logic

Elements of Logic:
Logical Sentences
Logical Entailment
Logical Proofs

Symbolic Logic
Problems with Natural Language
Benefits of Formal Language

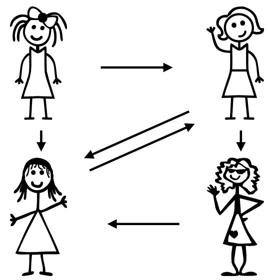
Course Logistics
Elements of Logic
Logics covered in the course
Advanced Logics *not* covered in the course

This lesson is an overview of the course. We start with a look at the basic elements of Logic - logical sentences, logical entailment, and logical proofs. We then see some of the problems that arise with natural language and see how those problems can be mitigated through the use of a formal language like the one we use in the course. And we conclude with some general remarks about Logic - a review of the basic elements, a summary of the different logics covered in the course, and a brief look at more advanced logics that are not covered.

Logical Sentences

For some people, Logic is an esoteric subject. They think it is used primarily by mathematicians in proving theorems in geometry or number theory. They think it is all about writing formal proofs to be published in scholarly papers that have little to do with everyday life. Nothing could be further from the truth.

Friends



As an example of using Logic in everyday life, consider the interpersonal relations of a small group of friends. There are just four members - Abby, Bess, Cody, and Dana. Some of the girls like each other, but some do not.

Friends

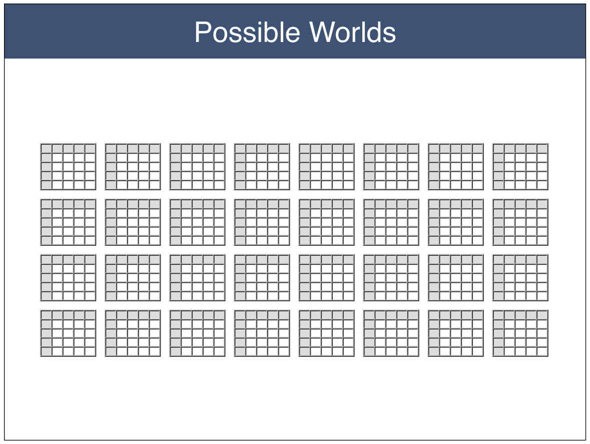
	Abby	Bess	Cody	Dana
Abby			✓	
Bess			✓	
Cody	✓	✓		✓
Dana			✓	

We can represent this information as a simple table like the one shown here. The checkmark in the first row here means that Abby likes Cody, while the absence of a checkmark means that Abby does not like the other girls (including herself). Bess likes Cody too. Cody likes everyone but herself. And Dana also likes the popular Cody.

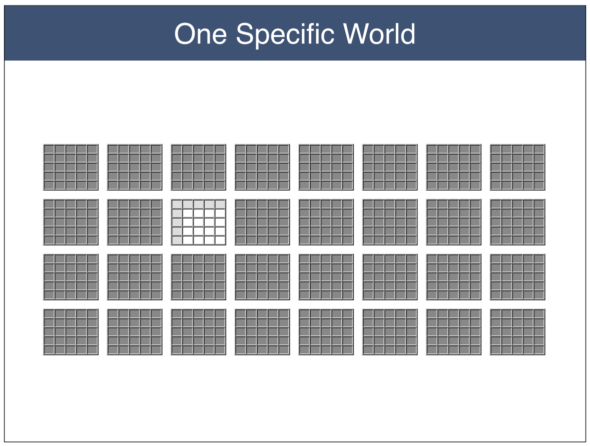
Friends

	Abby	Bess	Cody	Dana
Abby	✓		✓	
Bess		✓		✓
Cody	✓		✓	
Dana		✓		✓

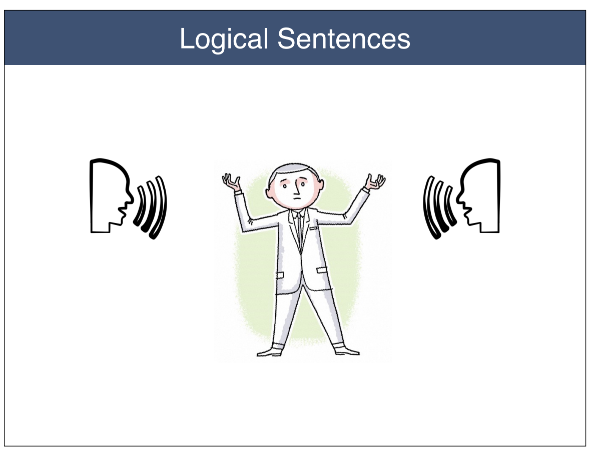
Of course, that is not the only possible state of affairs. This table shows another possible world. In this world, every girl likes exactly two other girls, and every girl is liked by just two girls.



Of course, there are more than two possible worlds. As it turns out, there are quite a few possibilities. Given four girls, there are sixteen possible instances of the likes relation - Abby likes Abby, Abby likes Bess, Abby likes Cody, Abby likes Dana, Bess likes Abby, and so forth. Each of these sixteen can be either true or false; there are 2^16 (65,536) combinations of these true-false possibilities; and so there are 2^16 possible worlds.



Of course, only one of these worlds is correct. However, we often do not know which world is correct.



Let's assume that we do not know the likes and dislikes of the girls ourselves but we have informants who are willing to tell us about them. Each informant knows a little about the likes and dislikes of the girls, but no one knows everything.

Logical Language

Dana likes Cody.

*Abby does **not** like Dana.*

*Dana does **not** like Abby.*

*Bess likes Cody **or** Dana.*

*Abby likes **everyone** that Bess likes.*

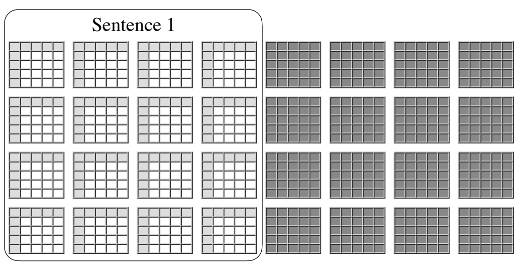
*Cody likes **everyone** who likes her.*

***No one** likes herself.*

This is where the language of Logic comes in. By writing logical sentences, each informant can express exactly what he or she knows - no more, no less. The sentences shown here are examples. The first sentence is straightforward; it tells us directly that Dana likes Cody. The second and third sentences tell us what is not true without directly saying what *is* true. The fourth sentence says that one condition holds or another but does not say which. The fifth sentence gives a general fact about the girls Abby likes. The sixth sentence expresses a general fact about Cody's likes. The last sentence says something about everyone.

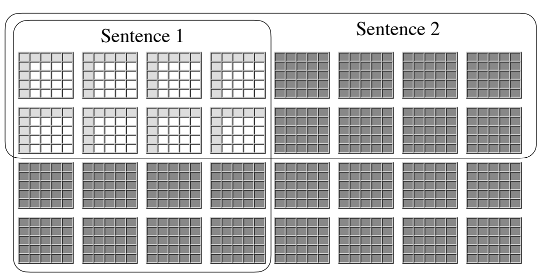
Looking at the worlds we saw earlier, we can see that all of these sentences are true in the first world. However, several of the sentences are false in the second world. Hence, we can rule out the second world.

After One Sentence



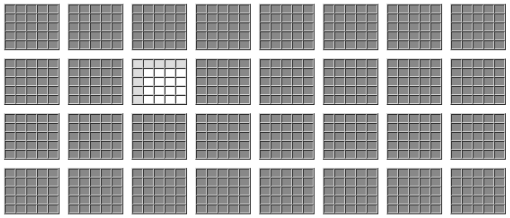
Logical sentences like the ones we saw earlier constrain the possible ways the world could be. Each sentence divides the set of possible worlds into two subsets, those in which the sentence is true and those in which the sentence is false, as suggested by the division here. Believing a sentence is tantamount to believing that the world is in the first set.

After Two Sentences



Given two sentences, we know the world must be in the intersection of the set of worlds in which the first sentence is true and the set of worlds in which the second sentence is true.

One Specific World



Ideally, when we have enough sentences, we know exactly how things stand.

Effective communication requires a language that allows us to express what we know, no more and no less. If we know the state of the world, then we should write enough sentences to communicate this to others. If we do not know which of various ways the world could be, we need a language that allows us to express only what we know. The beauty of Logic is that it gives us a means to express incomplete information when that is all we have and to express complete information when full information is available.

Complete Information

Premises:

Dana likes Cody.
Abby does not like Dana.
Dana does not like Abby.
Abby and Dana do not like Bess.
Bess likes Cody or Dana.
Abby likes everyone that Bess likes.
Cody likes everyone who likes her.
No one likes herself.

Conclusions:

Does Bess like Cody?
Does Abby like Bess?
Does Dana like Bess?

	Abby	Bess	Cody	Dana
Abby			✓	
Bess			✓	
Cody	✓	✓		✓
Dana			✓	

Once we know which world is correct, we can see that some sentences must be true even though they are not included in the premises we are given. For example, in the world shown here, we can see that Bess likes Cody, even though we are not told this fact explicitly. Similarly, we can see that Abby does not like Bess.

Logical Entailment

Logical Conclusions

Premises:

Dana likes Cody.
Abby does not like Dana.
Dana does not like Abby.
Abby and Dana do not like Bess.
Bess likes Cody or Dana.
Abby likes everyone that Bess likes.
Cody likes everyone who likes her.
No one likes herself.

Conclusions:

Bess likes Cody.
Abby does not like Bess.

	Abby	Bess	Cody	Dana
Abby			✓	
Bess			✓	
Cody	✓	✓		✓
Dana			✓	

Once we know which world is correct, we can see that some sentences must be true even though they are not included in the premises we are given. For example, in the world shown here, we can see that Bess likes Cody, even though we are not told this fact explicitly. Similarly, we can see that Abby does not like Bess.

Logical Conclusions

Premises:

Dana likes Cody.
Abby does not like Dana.
Dana does not like Abby.
Bess likes Cody or Dana.
Abby likes everyone that Bess likes.
Cody likes everyone who likes her.
No one likes herself.

	Abby	Bess	Cody	Dana
Abby			✓	
Bess			✓	
Cody	✓	✓		✓
Dana			✓	

	Abby	Bess	Cody	Dana
Abby		✓	✓	
Bess			✓	
Cody	✓	✓		✓
Dana			✓	

	Abby	Bess	Cody	Dana
Abby			✓	
Bess			✓	
Cody	✓	✓		✓
Dana		✓	✓	

	Abby	Bess	Cody	Dana
Abby		✓	✓	
Bess			✓	
Cody	✓	✓		✓
Dana		✓	✓	

Unfortunately, things are not always so simple. Although logical sentences can sometimes pinpoint a specific world from among many possible worlds, this is not always the case. Sometimes, a collection of sentences only partially constrains the world. For example, there are four different worlds that satisfy the premises shown here.

Logical Conclusions

Premises:

Dana likes Cody.
Abby does not like Dana.
Dana does not like Abby.
Bess likes Cody or Dana.
Abby likes everyone that Bess likes.
Cody likes everyone who likes her.
No one likes herself.

Questions:

Does Bess like Cody?
Does Abby like Bess?
Does Dana like Bess?

	Abby	Bess	Cody	Dana
Abby			✓	
Bess			✓	
Cody	✓	✓		✓
Dana			✓	

	Abby	Bess	Cody	Dana
Abby		✓	✓	
Bess			✓	
Cody	✓	✓		✓
Dana			✓	

	Abby	Bess	Cody	Dana
Abby			✓	
Bess			✓	
Cody	✓	✓		✓
Dana		✓	✓	

	Abby	Bess	Cody	Dana
Abby		✓	✓	
Bess			✓	
Cody	✓	✓		✓
Dana		✓	✓	

In situations like this, which world should we use in answering questions? The good news it that sometimes it does not matter. Even though a set of sentences does not determine a unique world, there are some sentences that have the same truth value in *every* world that satisfies the given sentences, and we can use that value in answering questions.

Logical Conclusions

Premises:
Dana likes Cody.
Abby does not like Dana.
Dana does not like Abby.
Bess likes Cody or Dana.
Abby likes everyone that Bess likes.
Cody likes everyone who likes her.
No one likes herself.

Questions:
Does Bess like Cody? Yes
Does Abby like Bess? No
Does Dana like Bess? Maybe

	Abby	Bess	Cody	Dana
Abby			✓	
Bess			✓	
Cody	✓	✓		✓
Dana			✓	

	Abby	Bess	Cody	Dana
Abby		✓	✓	
Bess			✓	
Cody	✓	✓		✓
Dana			✓	

	Abby	Bess	Cody	Dana
Abby			✓	
Bess			✓	
Cody	✓	✓		✓
Dana		✓	✓	

	Abby	Bess	Cody	Dana
Abby		✓	✓	
Bess			✓	
Cody	✓	✓		✓
Dana		✓	✓	

In our example, we see that Bess likes Cody in all four worlds, so the answer to the first question here is Yes. We also see Bess does *not* like Abby, so the answer to the second question is No. Of course, there are cases where different possible worlds have different values. Does Dana like Bess? Since there are different values in different worlds, we cannot say yes and we cannot say no. All we can say is Maybe. In the real world, either Dana likes Bess or she doesn't. However, we do not have enough information to say which case is correct.

Logical Entailment

A set of premises *logically entails* a conclusion if and only if *every* world that satisfies the premises satisfies the conclusion.

This is *logical entailment*. We say that a set of premises logically entails a conclusion if and only if every world that satisfies the premises also satisfies the conclusion.

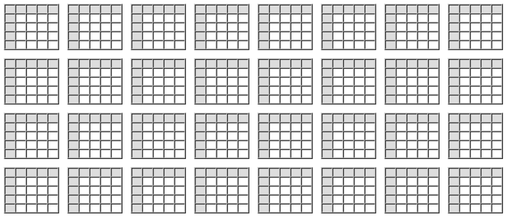
Model Checking

Iterate through *all* possible worlds. For every world that satisfies the premises, check if it satisfies the conclusion.

Model checking is the process of examining the set of all possible worlds to determine logical entailment. To check whether a set of sentences logically entails a conclusion, we use our premises to determine which worlds are possible and then check the possible worlds to see whether or not they satisfy our conclusion. If the number of possible worlds is not too large, this method works well.

Problem with Model Checking

Lots of Worlds (sometimes infinitely many)



Model Checking is like solving polynomial equations by enumerating all possible values for the variables.

Unfortunately, in general, there are many, many possible worlds; and, in some cases, the number of possible worlds is infinite, in which case model checking is impossible. So what do we do?

Logical Proofs

The answer is logical reasoning and logical proofs.

Symbolic Manipulation in Algebra

$$x - 3y = 0$$
$$x + y = 12$$
$$\downarrow$$
$$-4y = -12$$
$$\downarrow$$
$$y = 3$$
$$x = 9$$

Logical proofs are analogous to derivations in algebra. To illustrate this analogy, consider the algebra problem shown here. We know that x-3y is 0 and x+y is 12. We could try solving these equations by randomly trying different values for x and y. However, we can get to an answer faster by manipulating our equations syntactically. First we subtract the second equation from the first and get -4y = -12. Then, we divide each side -4 to get a value for y. And finally. substituting back into one of the preceding equations, we get a value for x.

Symbolic Manipulation in Logic

Dana likes Cody.
Abby does not like Dana.
Dana does not like Abby.
Abby and Dana do not like Bess.
Bess likes Cody or Dana.
Abby likes everyone that Bess likes.
Cody likes everyone who likes her.
No one likes herself.

↓

Bess likes Cody.
Abby does not like Bess.

Logical reasoning is similar. Rather than checking possible worlds, we apply syntactic operations to the sentences we are given to generate new sentences.

Rules of Inference

A *rule of inference* is a reasoning pattern consisting of some premises and some conclusions.

A *proof* is a sequence of sentences in which every sentence is either a premise or the result of applying a *rule of inference* to earlier elements of the sequence.

One of Aristotle's great contributions to philosophy was the identification of operations of this sort. Such operations are typically called rules of inference. By applying rules of inference to premises, we produce conclusions that are entailed by those premises. A proof is simply a sequence of such rule applications. We can think of individual reasoning steps as the *atoms* out of which logical proof *molecules* are built.

Sample Rule of Inference

All *of Abby's friends are Bess's friends.*
All *of Bess's friends are Cody's friends.*
Therefore, ***all*** *of Abby's friends are Cody's friends.*

As an example of a rule of inference, consider the reasoning step shown below. We know that all of Abby's friends are Bess's friends, and we know that all of Bess's friends are Cody's friends. Consequently, we can conclude that all of Abby's friends are Cody's friends.

Sample Rule of Inference

All *Accords are Hondas.*
All *Hondas are Japanese.*
Therefore, ***all*** *Accords are Japanese.*

Here is another example. All Accords are Hondas, and all Hondas are Japanese. Consequently, we can conclude that all Accords are Japanese.

Sample Rule of Inference

All *borogoves are slithy toves.*
All *slithy toves are mimsy.*
Therefore, ***all*** *borogoves are mimsy.*

And yet another example. If we know that all borogoves are slithy toves and we know that all slithy toves are mimsy, then we can conclude that all borogoves are mimsy. What's more, in order to reach this conclusion, we do not need to know anything else about borogoves or slithy toves or what it means to be mimsy.

General Rule of Inference

All *x are y.*
All *y are z.*
Therefore, ***all*** *x are z.*

What is interesting about these examples is that they share the same reasoning structure, viz. the pattern shown here. ... Two things are worthy of note here. First of all, correctness in logical reasoning is determined by the logical operators in our sentences, not the objects and relationships mentioned in those sentences. Second, the conclusion is guaranteed to be true only if the premises are true.

Bertrand Russell

Logic "may be defined as the subject in which we never know what we are talking about nor whether what we are saying is true."

- Bertrand Russell

The philosopher Bertrand Russell summed this situation up in this quote. Logic "may be defined as the subject in which we never know what we are talking about nor whether what we are saying is true." We do not need to know anything about the concepts in our premises except for the information captured in those premises. Our conclusion must be true is our premises are true; but it can be false if one or more of our premises is false.

Unsound Rule of Inference

***All** x are y.*
***Some** y are z.*
*Therefore, **some** x are z.*

No! No!! No!!!

The existence of reasoning patterns is fundamental in Logic but raises important questions. Which patterns are correct? Are there many such patterns or just a few?

Let us consider the first of these questions. Obviously, there are patterns that are just plain wrong in the sense that they can lead to incorrect conclusions. Consider, as an example, the *faulty* reasoning pattern shown here.

Using Unsound Rule of Inference

***All** Toyotas are Japanese cars.*
***Some** Japanese cars are made in America.*
*Therefore, **some** Toyotas are made in America.*

Sometimes produces a result that *happens* to be true.

Now let us take a look at an instance of this pattern. If we replace x by Toyotas and y by Japanese and z by made in America, we get the following line of argument, leading to a conclusion that happens to be correct.

Using Unsound Rule of Inference

All *Toyotas are cars.*
Some *cars are Porsches.*
Therefore, ***some*** *Toyotas are Porsches.*

Sometimes produces a result that *happens* to be false.

On the other hand, if we replace x by Toyotas and y by cars and z by Porsches, we get a line of argument leading to a conclusion that is not correct.

Deduction

A rule of inference is *sound* if and only if the conclusion is true whenever the premises are true.

The application of sound rules of inference is called *deduction*.

What distinguishes a correct pattern from one that is incorrect is that it must always lead to correct conclusions, i.e. conclusions that are logically entailed by the premises. As we will see, this is the defining criterion for what we call *deduction*. Of all types of reasoning, deductive reasoning is the only one that guarantees its conclusions in all cases. It has some very special properties and holds a unique place in Logic.

Induction

Induction is reasoning from the specific to the general.

I have seen 1000 black ravens.
I have never seen a raven that is not black.
Therefore, every raven is black.

If induction is incomplete, it is not necessarily sound (but it can be useful).

Now, it is noteworthy that there are patterns of reasoning that are not always correct but are sometimes useful. There is induction, abduction, analogy, and so forth.

Induction is reasoning from the particular to the general. The example shown below illustrates this. In this case the induction is incomplete. Although we have seen many ravens, we have not seen them all. However, if we see enough cases in which something is true and we never see a case in which it is false, we tend to conclude that it is always true. Unfortunately, when induction is incomplete, as in this case, it is not sound. There might be an albino raven that we have not yet seen.

Induction versus Deduction

Induction is the basis for **Science** (and machine learning)
Deduction is the subject matter of **Logic**.

Science aspires to discover / propose **new** knowledge.
Logic aspires to apply and/or analyze **existing** knowledge.

Incomplete induction is the basis for Science (and machine learning). Deduction is the subject matter of Logic. Science aspires to discover / propose new knowledge. Logic aspires to apply and/or analyze existing knowledge.

Niels Bohr to Albert Einstein

"You are not thinking; you are just being logical."

This distinction was at the heart of a disagreement between the physicist Albert Einstein and his contemporary Niels Bohr in which Bohr derided Einstein's emphasis on deduction rather than induction. He reputedly told Einstein: "You are not thinking; you are just being logical." Bohr was a fan of induction and thought that Einstein placed too much emphasis on deduction.

Entailment versus Provability

A set of premises *logically entails* a conclusion if and only if every world that satisfies the premises satisfies the conclusion.

A conclusion is *provable* from a set of premises if and only if there is a finite sequence of sentences in which every element is either a premise or the result of applying a *sound* rule of inference to earlier members in the sequence.

In talking about Logic, we now have two notions - logical entailment and provability. A set of premises logically entails a conclusion if and only if every possible world that satisfies the premises also satisfies the conclusion. A sentence is provable from a set of premises if and only if there is a finite proof of the conclusion from the premises. The concepts are quite different. One is based on possible worlds; the other is based on symbolic manipulation of expressions. Yet, for the proof systems we will be seeing, they are closely related.

Soundness and Completeness

As we shall see, for well-behaved logics, logical entailment and provability are identical - a set of premises **logically entails** a conclusion *if and only if* the conclusion is **provable** from the premises.

This is a very big deal.

Although these two notions are defined in different ways, for well-behaved logics, the two notions turn out to be the same - a set of premises logically entails a conclusion if and only if the conclusion is provable from the premises. 2+2 = 2*2. Morning star = evening star.

This is a very big deal. It means that we can do logical reasoning in two very different ways, choosing whichever is most convenient; and we can rest assured that we will get the same result whichever way we choose.

Symbolic Logic

Logical Sentences

Dana likes Cody.

*Abby does **not** like Dana.*

*Dana does **not** like Abby.*

*Bess likes Cody **or** Dana.*

*Abby likes **everyone** that Bess likes.*

*Cody likes **everyone** who likes her.*

***Everyone** likes herself.*

In the examples we have seen thus far, the sentences were written in English. While natural language works well to express logical information in many circumstances, it is not without its problems. Natural language sentences can be complex; they can be ambiguous; and failing to understand the meaning of a sentence can lead to errors in reasoning.

Complexity of Natural Language

One grammatically correct sentence:

The cherry blossoms in the spring.

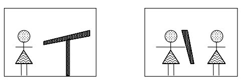
Another grammatically correct sentence:

The cherry blossoms in the spring sank.

Even very simple sentences can be complex. Here we see two grammatically legal sentences. They are the same in all but the last word, but there structure is entirely different. In the first, the main verb is blossoms, while in the second blossoms is a noun and the main verb is sank.

Grammatical Ambiguity

There's a girl in the room with a telescope.



As an example of ambiguity, suppose I were to write the sentence There's a girl in the room with a telescope. Here we see two possible meanings of this sentence. Am I saying that there is a girl in a room containing a telescope? Or am I saying that there is a girl in the room and she is holding a telescope?

Newseum Headlines

Crowds Rushing to See Pope Trample 6 to Death

Such complexities and ambiguities can sometimes be humorous if they lead to interpretations the author did not intend, as in various infamous newspaper headlines with multiple interpretations.

Crowds Rushing to See Pope Trample 6 to Death. (Who would have guessed that the pope could be so cruel?).

Newseum Headlines

Crowds Rushing to See Pope Trample 6 to Death

Scientists Grow Frog Eyes and Ears

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Newseum Headlines

Crowds Rushing to See Pope Trample 6 to Death

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Fried Chicken Cooked in Microwave Wins Trip

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British Left Waffles on Falkland Islands

British Left Waffles on Falkland Islands. (Are those in the British left unclear on what to do or do the Brits simply not like waffles?)

Newseum Headlines

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Indian Ocean Talks

Indian Ocean Talks. (I always imagine it has a deep voice.)

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Using a formal language eliminates such unintentional ambiguities (and, for better or worse, it avoids any unintentional humor as well).

Misteaks in Print

Residents report that a hole was cut in the fence surrounding a nudist colony. Police are ***looking into*** *it.*

Such ambiguities are not limited to headlines. For example, we have this quote from the text of a newspaper article. *Residents report that a hole was cut in the fence surrounding a nudist colony. Police are looking into it.*

Doug Lenat's Logic

Champagne is better than beer.
Beer is better than soda.
***Therefore**, champagne is better than soda.*

And the problems are not limited to ambiguity. As an example, consider the rule shown here. *Champagne is better than beer. Beer is better than soda. Therefore, champagne is better than soda.* That seems reasonable.

Doug Lenat's Logic

Champagne is better than beer.
Beer is better than soda.
***Therefore**, champagne is better than soda.*

X is better than Y.
Y is better than Z.
***Therefore**, X is better than Z.*

It is a special case of transitivity. *X is better than Y. Y is better than Z. Therefore, X is better than Z.*

Doug Lenat's Logic

Champagne is better than beer.
Beer is better than soda.
***Therefore**, champagne is better than soda.*

X is better than Y.
Y is better than Z.
***Therefore**, X is better than Z.*

Bad sex is better than nothing.
Nothing is better than good sex.
***Therefore**, bad sex is better than good sex.*
Really?

Now consider what happens when we apply the same transitivity rule in the case illustrated here. *Bad sex is better than nothing, and nothing is better than good sex; therefore, bad sex is better than good sex.* The form of the argument is the same as before, but the conclusion is somewhat less believable. The problem in this case is that the use of the word "nothing" here is syntactically similar to the use of beer in the preceding example, but in English it means something entirely different.

Logic eliminates difficulties like this through the use of a formal language for encoding information. Given the syntax and semantics of this formal language, we can give a precise definition for the notion of logical conclusion. Moreover, we can establish precise reasoning rules that produce all and only logical conclusions.

Conclusion

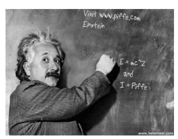
Elements of Logic

Logical Language

Observations: *Abby likes Bess but Bess does not like Abby.*
Definitions: *A bachelor is an unmarried male*.
Laws: *Parents are older than their children*.

Model Checking

Logical Proofs



To recap, in this course, we talk about three separate elements of logic and their interrelationships. We introduce a formal language to express record our observations, to define concepts, to formalize physical and artificial laws. We see how to use logical reasoning to derive conclusions from these bits of information. And we see how to construct logical proofs to convince others of our conclusions.

Multiple Logics

Propositional Logic (logical operators)

If *it is raining* ***and*** *it is cold*, ***then*** *the ground is wet*.

Although Logic is a single field of study, there is more than one logic. In the three main units of this course, we look at three different types of logic.

Propositional Logic is the logic of propositions. Symbols in the language represent "conditions" in the world, and complex sentences in the language express interrelationships among these conditions. The primary operators are Boolean connectives, such as and, or, and not.

Multiple Logics

Propositional Logic (logical operators)

If *it is raining* ***and*** *it is cold,* ***then*** *the ground is wet.*

Relational Logic (variables and quantifiers)

If ***x*** *is a parent of* ***y****, then* ***x*** *is older than* ***y****.*

Relational Logic expands upon Propositional Logic by providing a means for explicitly talking about objects and their interrelationships (not just true-false conditions). In order to do so, we expand our language to include object constants, function constants, relation constants, variables, and quantifiers.

Multiple Logics

Propositional Logic (logical operators)

If *it is raining* ***and*** *it is cold,* ***then*** *the ground is wet.*

Relational Logic (variables and quantifiers)

If ***x*** *is a parent of* ***y****, then* ***x*** *is older than* ***y****.*

Term Logic (compound terms)

{a, b} is a subset of {a, b, c}.

Term Logic takes us one step further by providing a means for describing worlds with infinitely many objects. The resulting logic is much more powerful than Propositional Logic and Relational Logic. Unfortunately, as we shall see, some of the nice computational properties of the first two logics are lost as a result.

Each logic introduces new issues and capabilities. Despite these differences, there are many commonalities among these logics. In particular, in each case, there is a language with a formal syntax and a precise semantics; there is a notion of logical entailment; and there are legal rules for manipulating expressions in the language. These similarities allow us to compare the various logics and to gain an appreciation of the tradeoffs between expressiveness and computational complexity.

Logical Extensions

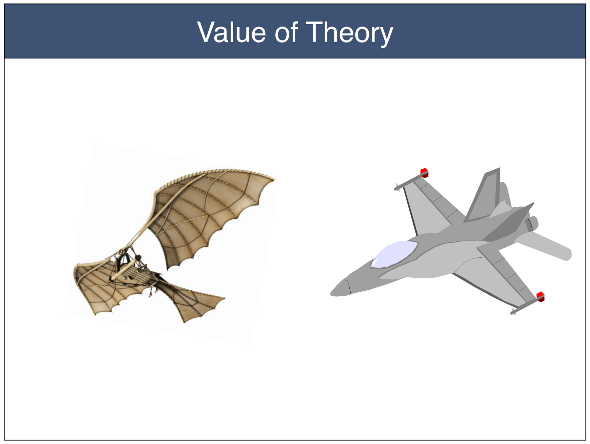
Language

Probabilities
Metaknowledge - knowledge about knowledge
Paradoxes, e.g. *This sentence is false.*

Reasoning

Negation as Failure - *knowing not* versus *not knowing*
Induction, Abduction, Analogical Reasoning
Paraconsistent Reasoning - reasoning with inconsistency

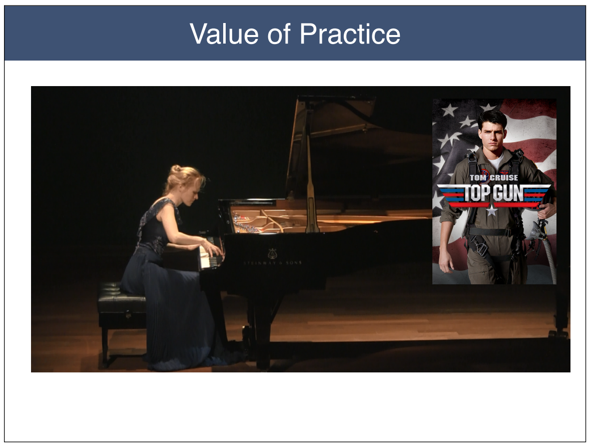
There are also some topics that are relevant to logic but are out of scope for this course, such as probability, metaknowledge, and paradoxes. Also, negation as failure, non-deductive reasoning methods (like induction), and paraconsistent reasoning. We touch on these extensions in this course, but we do not talk about them in any depth.



The importance of Logic in our lives raises the question of how we come by the ability to use Logic. To some extent, it happens without any conscious effort, like the ability to recognize faces. However, some elements of Logic need to be taught explicitly, in the same way that we teach arithmetic and algebra.

If we built airplanes by emulating what we see in nature, we would end up with machines that flap their wings (so-called ornithopters). By studying fluid dynamics and engineering, it is possible to build supersonic aircraft that perform much better.

The situation with Logic is similar. We can do limited logical reasoning without any formal training. However, studying Logic as a subject in its own right allows us to move beyond the simple types of reasoning that we pick up on our own and learn to use techniques that we might not otherwise know about.



In addition, there is the benefit of practice. Just knowing how to read notes does not make one a concert pianist. That takes practice. Just knowing about a logical reasoning technique does not make one a master logician. That also takes practice. Note that, in the movie Top Gun, even accomplished fighter pilots are sent back to school - to learn new skills and to practice those skills. The same works for Logic. To quote Tom Skerritt in Top Gun: "You're the best of the best. We'll make you better."