Introduction
Logic

*Logic* is the study of information encoded in the form of logical sentences.

Sacramento is the capital of California.
Boise is *not* the capital of Utah.
Boise is the capital of Utah *or* Idaho.
If Eugene is not the capital of Oregon, *then* it is Salem.
There is *some* city that is capital of Hawaii.
Every *state* has a capital.
Logic in Human Affairs

Language of Logic

*Abby likes Bess.*

*A triangle is a polygon with three sides.*

*Force equal mass times acceleration.*

Logical Reasoning

to derive conclusions

to convince others
Logic-Enabled Computer Systems

Email Readers

*If the message is from “genesereth” and the topic is “logic”, Then the message goes in the “important” folder*

eCommerce Systems

*If the product is a notebook and the customer is a student and the date is in December, Then the price of 5.99.*
Logic Programming

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\begin{align*}
\text{init}(\text{cell}(1,1,b)) & \quad \text{init}(\text{cell}(1,2,b)) \\
\text{init}(\text{cell}(1,3,b)) & \quad \text{init}(\text{cell}(2,1,b)) \\
\text{init}(\text{cell}(2,2,b)) & \quad \text{init}(\text{cell}(2,3,b)) \\
\text{init}(\text{cell}(3,1,b)) & \quad \text{init}(\text{cell}(3,2,b)) \\
\text{init}(\text{cell}(3,3,b)) & \quad \text{init}(\text{control}(x))
\end{align*}
\]

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\begin{align*}
\text{legal}(P, \text{mark}(X,Y)) :&= \quad \text{legal}(x, \text{noop}) :\quad \text{legal}(o, \text{noop}) : \\
\text{true}(\text{cell}(X,Y,b)) & \quad \text{true}(\text{control}(P)) \\
\text{true}(\text{control}(x)) & \quad \text{true}(\text{control}(o)) \\
\text{true}(\text{control}(x)) & \quad \text{true}(\text{control}(o))
\end{align*}
\]

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\begin{align*}
\text{next}(\text{cell}(M,N,P)) :&= \quad \text{next}(\text{cell}(M,N,Z)) : \\
\text{does}(P, \text{mark}(M,N)) & \quad \text{does}(P, \text{mark}(M,N)) & \quad \text{true}(\text{cell}(M,N,Z)) & \quad Z\#b \\
\text{next}(\text{cell}(M,N,b)) :&= \quad \text{next}(\text{cell}(M,N,b)) : \\
\text{does}(P, \text{mark}(J,K)) & \quad \text{true}(\text{cell}(M,N,b)) & \quad (M\#J \mid N\#K) \\
\text{next}(\text{control}(x)) :&= \quad \text{next}(\text{control}(o)) : \\
\text{true}(\text{control}(x)) & \quad \text{true}(\text{control}(o)) \\
\text{true}(\text{control}(x)) & \quad \text{true}(\text{control}(o))
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\text{row}(M,P) :&= \quad \text{column}(N,P) : \\
\text{true}(\text{cell}(M,1,P)) & \quad \text{true}(\text{cell}(M,2,P)) & \quad \text{true}(\text{cell}(M,3,P)) \\
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\end{align*}
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\text{goal}(x,100) :&= \quad \text{goal}(x,50) : \\
\text{line}(x) & \quad \text{draw} \\
\text{goal}(x,0) :&= \quad \text{line}(o) \\
\text{line}(x) & \quad \text{line}(o)
\end{align*}
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\begin{align*}
\text{goal}(o,100) :&= \quad \text{goal}(o,50) : \\
\text{line}(o) & \quad \text{draw} \\
\text{goal}(o,0) :&= \quad \text{line}(x) \\
\text{line}(o) & \quad \text{line}(x)
\end{align*}
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\text{goal}(o,100) :&= \quad \text{draw} : \\
\text{line}(o) & \quad \text{line}(x) & \quad \text{~line}(o)
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\text{goal}(o,50) :&= \quad \text{draw} : \\
\text{line}(o) & \quad \text{line}(x) & \quad \text{~line}(o)
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\text{goal}(o,0) :&= \quad \text{line}(x) \\
\text{line}(o) & \quad \text{~line}(x) & \quad \text{~line}(o)
\end{align*}
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\text{goal}(100) :&= \quad \text{draw} : \\
\text{line}(x) & \quad \text{line}(o) & \quad \text{~line}(o)
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\text{goal}(50) :&= \quad \text{draw} : \\
\text{line}(x) & \quad \text{line}(o) & \quad \text{~line}(o)
\end{align*}
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\begin{align*}
\text{goal}(0) :&= \quad \text{line}(x) \\
\text{line}(o) & \quad \text{~line}(x) & \quad \text{~line}(o)
\end{align*}
\]

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\begin{align*}
\text{terminal} :&= \quad \text{line}(P) \\
\text{~open}
\end{align*}
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\begin{align*}
\text{diagonal}(P) :&= \quad \text{diagonal}(P) : \\
\text{true}(\text{cell}(1,1,P)) & \quad \text{true}(\text{cell}(2,2,P)) & \quad \text{true}(\text{cell}(3,3,P)) \\
\text{true}(\text{cell}(1,3,P)) & \quad \text{true}(\text{cell}(2,2,P)) & \quad \text{true}(\text{cell}(3,1,P))
\end{align*}
\]

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\begin{align*}
\text{line}(P) :&= \quad \text{row}(M,P) : \\
\text{line}(P) & \quad \text{column}(N,P) & \quad \text{diagonal}(P)
\end{align*}
\]

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\begin{align*}
\text{open} :&= \quad \text{true}(\text{cell}(M,N,b)) \\
\text{draw} & \quad \text{~line}(x) & \quad \text{~line}(o)
\end{align*}
\]
Elements of Logic
Topics

Logical Language
  Logical expressions
  Meaning of those expressions

Logical Entailment
  Given sentences we know to be true, what other sentences must also be true?

Symbolic Manipulation
  Rules for syntactically manipulating expressions to derive those conclusions
<table>
<thead>
<tr>
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<th>Abby</th>
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Logical Sentences

Dana likes Cody.
Abby does not like Dana.
Dana does not like Abby.
Bess likes Cody or Dana.
Abby likes everyone that Bess likes.
Cody likes everyone who likes her.
Nobody likes herself.
### Possible Worlds

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Logical Entailment

A set of premises *logically entails* a conclusion if and only if every world that satisfies the premises satisfies the conclusion.

**Premises:**
- Dana likes Cody.
- Abby does not like Dana.
- Dana does not like Abby.
- Bess likes Cody or Dana.
- Abby likes everyone that Bess likes.
- Cody likes everyone who likes her.
- Nobody likes herself.

**Conclusions:**
- Bess likes Cody.
- Bess does not like Dana.
- Everybody likes somebody.
- Everybody is liked by somebody.
## Checking Possible Worlds

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Proof

We know that Abby likes everyone that Bess likes, and we know that Abby does not like Dana. Therefore, Bess must not like Dana either. (If Bess did like Dana, then Abby would like her as well.) At the same time, we know that Bess likes Cody or Dana. Consequently, since Bess does not like Dana, she must like Cody.
Rules of Inference

A *rule of inference* is a reasoning pattern consisting of some premises and some conclusions.

If we believe the premises, a rule of inference tell us that we should also believe the conclusions.
Sample Rule of Inference

All Accords are Hondas.
All Hondas are Japanese.

Therefore, all Accords are Japanese.
Sample Rule of Inference

All borogoves are slithy toves.
All slithy toves are mimsy.

Therefore, all borogoves are mimsy.
Sound Rule of Inference

All $x$ are $y$.
All $y$ are $z$.

*Therefore*, $x$ are $z$.

Which patterns are correct?
How many rules do we need?
Bad Rule of Inference

All $x$ are $y$.

Some $y$ are $z$.

Therefore, some $x$ are $z$. 


Using Bad Rule of Inference

All Toyotas are Japanese cars.
Some Japanese cars are made in America.
Therefore, some Toyotas are made in America.
Using Bad Rule of Inference

All Toyotas are cars.
Some cars are Porsches.
Therefore, some Toyotas are Porsches.
Formalization
Example of Complexity

One grammatically correct sentence:

*The cherry blossoms in the spring.*

Another grammatically correct sentence:

*The cherry blossoms in the spring sank.*
Michigan Lease Termination Clause

The University may terminate this lease when the Lessee, having made application and executed this lease in advance of enrollment, is not eligible to enroll or fails to enroll in the University or leaves the University at any time prior to the expiration of this lease, or for violation of any provisions of this lease, or for violation of any University regulation relative to resident Halls, or for health reasons, by providing the student with written notice of this termination 30 days prior to the effective date of termination; unless life, limb, or property would be jeopardized, the Lessee engages in the sales of purchase of controlled substances in violation of federal, state or local law, or the Lessee is no longer enrolled as a student, or the Lessee engages in the use or possession of firearms, explosives, inflammable liquids, fireworks, or other dangerous weapons within the building, or turns in a false alarm, in which cases a maximum of 24 hours notice would be sufficient.
Grammatical Ambiguity

There’s a girl in the room with a telescope.
Crowds Rushing to See Pope Trample 6 to Death

British Left Waffles on Falkland Islands

Scientists Grow Frog Eyes and Ears

Food Stamp Recipients Turn to Plastic

Fried Chicken Cooked in Microwave Wins Trip

Indian Ocean Talks
Reasoning

Champagne is better than beer.
Beer is better than soda.

Therefore, champagne is better than soda.
Champagne is better than beer.
Beer is better than soda.

Therefore, champagne is better than soda.

Bad sex is better than nothing.
Nothing is better than good sex.

Therefore, bad sex is better than good sex.
Formal Logic

Simple Syntax
   Easy to read
   Grammatically unambiguous

Clear Semantics
   Tells us what each sentence says
   Tells us which conclusions follow from premises

Precise Rules of Inference
   Each rule is sound
   Rules are complete
Algebra Problem

Xavier is three times as old as Yolanda. Xavier's age and Yolanda's age add up to twelve. How old are Xavier and Yolanda?
Algebra Solution

Xavier is three times as old as Yolanda. Xavier's age and Yolanda's age add up to twelve. How old are Xavier and Yolanda?

\[
\begin{align*}
  x - 3y &= 0 \\
  x + y &= 12 \\
  -4y &= -12 \\
  y &= 3 \\
  x &= 9
\end{align*}
\]
Logic Problem

If Mary loves Pat, then Mary loves Quincy. If it is Monday and raining, then Mary loves Pat or Quincy. If it is Monday and raining, does Mary love Quincy?

If it is Monday and raining, does Mary love Pat?

Mary loves only one person at a time. If it is Monday and raining, does Mary love Pat?
Formalization

Symbols:

Mary loves Pat. \( p \quad \) It is Monday. \( m \)

Mary loves Quincy. \( q \quad \) It is raining. \( r \)

Premises:

\[ p \Rightarrow q \]
\[ m \land r \Rightarrow p \lor q \]

Question:

\[ m \land r \Rightarrow q? \]
Logic Problem Revisited

If Mary loves Pat, then Mary loves Quincy. If it is Monday and raining, then Mary loves Pat or Quincy. If it is Monday raining, does Mary love Quincy?

\[ p \Rightarrow q \]
\[ m \land r \Rightarrow p \lor q \]

\[ m \land r \Rightarrow q \lor \neg q \]
\[ m \land r \Rightarrow q \]
Automation
Automated Reasoning

\[ p(a,b) \quad q(b,c) \]

\[ \neg p(b,d) \quad \forall x. \forall y. (p(x,y) \implies q(x,y)) \]

\[ p(c,b) \lor p(c,d) \quad \exists x. p(x,d) \]
Logic Technology

Languages
Knowledge Interchange Format (KIF) - ANSI
Common Logic - W3C

Some Popular Automated Reasoning Systems
Otter / Snark / Vampire / …
PTTP / Epilog

Knowledge Bases
Definitions (Bachelor is an unmarried adult male.)
Physical Laws (e.g. $PV=nRT$)
Laws (e.g. 1040 necessary if earnings > $n$.)
Mathematics

Group Axioms

\[(x \times y) \times z = x \times (y \times z)\]
\[x \times e = x\]
\[e \times x = x\]
\[x \times x^{-1} = e\]

Theorem

\[x^{-1} \times x = e\]

Tasks:

Proof Checking

Proof Generation
Some Successes

Various Theorems

- 4 color theorem (Appel and Haken)
- the limit of a sum is the sum of the limits
- the Bolzano-Weierstrass Theorem
- the Fundamental Theorem of calculus
- Euler's identity
- Gauss' law of quadratic reciprocity
- the undecidability of the halting problem
- Godel's incompleteness theorem (Shankar)

Other

- Thousands of Problems for Theorem Provers (TPTP)
- CADE ATP Systems Competition (CASC)
Hardware Engineering

Circuit:

Premises:

- $o \iff (x \land \neg y) \lor (\neg x \land y)$
- $a \iff z \land o$
- $b \iff x \land y$
- $s \iff (o \land \neg z) \lor (\neg o \land z)$
- $c \iff a \lor b$

Applications:
- Simulation
- Design
- Diagnosis
- Test Generation

Conclusion:

- $x \land y \Rightarrow \neg c$
# Deductive Database Systems

## Database Tables

**parent**

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<td>parent(bob, coe)</td>
</tr>
</tbody>
</table>

## Virtual tables

\[ parent(x, y) \land parent(y, z) \Rightarrow grandparent(x, z) \]

## Constraints

\[ parent(x, x) \Rightarrow illegal \]
\[ parent(x, y) \land parent(y, x) \Rightarrow illegal \]
Logical Spreadsheets
Examples of Logical Constraints

Scheduling
Start times must be before end times
Room 104 may not be scheduled after 5:00 pm
Only senior managers can reserve the third floor conference room

Travel Reservations
The number of lap infants in a group on a flight must not exceed the number of adults.

Academic Programs
Students must take at least 2 math courses
Regulations and Business Rules

Using the language of logic, it is possible to define new relations.

Office mates are people who share an office.

\[ \text{office}(x,z) \land \text{office}(y,z) \Rightarrow \text{officemate}(x,y) \]

This includes the property of legality / illegality.

Managers and subordinates may not be office mates.

\[ \text{manages}(x,y) \land \text{officemate}(x,y) \Rightarrow \text{illegal} \]
Study Guide
Symbolic Manipulation

\[
x - 3y = 0 \\
x + y = 12 \\
\frac{-4y = -12}{-4}
\]
Mathematical Background

Sets

\{a, b, c\} \cup \{b, c, d\} = \{a, b, c, d\}

a \in \{a, b, c\}

\{a, b, c\} \subseteq \{a, b, c, d\}

Functions and Relations

\text{r} (a, b, c)

f(a, b) = c
Hints on How to Take the Course

Materials of the Course
  Lectures
  Lecture Notes
  Additional Readings
  Exercises

Discussion Groups
  Read discussion
  Post questions
  Answer questions
  Work with others!
Multiple Logics

Propositional Logic

*If it is raining, the ground is wet.*

Relational Logic

*If x is a parent of y, then y is a child of x.*

Herbrand Logic

*The father of a person is older than the person.*
Meta

We will frequently write sentences about sentences.

Sentence: *When it rains, it pours.*
Metasentence: *That sentence contains two verbs.*

We will frequently prove things about proofs.

Proofs: formal
Metaproofs: informal