# CS 157 Quiz 1 Review October 17<sup>th</sup> 2023

# Agenda

- 1. Validity, Unsatisfiability, Contingency
- 2. Logical Entailment, Equivalence, Consistency
- 3. Soundness and Completeness
- 4. Hilbert Proofs
- 5. Natural Deduction & Fitch Proofs
- 6. Resolution

# Validity, Unsatisfiability, Contingency

# Propositional Logic & Analysis

 $\rightarrow$  Valid: Satisfied by every truth assignment.

- E.g. (р ∨ ¬р).
- Always yields T in a truth table
- $\rightarrow$  **Unsatisfiable**: Not satisfied by *any* truth assignment.
  - E.g. (р ∧ ¬р)
  - Always yields F in a truth table

 $\rightarrow$  **Contingent:** There is some truth assignment that satisfies it, and some truth assignment that falsifies it

- E.g. ( $p \land q$ ): If p and q both true, it is true. If p and q both false, it is false

# Logical Entailment, Equivalence, Consistency

# Logical Entailment

- A set of sentences logically entails a sentence iff every truth assignment that satisfies the premises also satisfies the conclusion
- Example 1:
  - $\{p \Rightarrow r\} \vDash (p \Rightarrow q \lor r)$
- Example 2:
  - $\{p \Rightarrow q \lor r, p \Rightarrow r\} \vDash (q \Rightarrow r)$
- Extra Practice:
  - http://intrologic.stanford.edu/exercises/exercise\_03\_03.html
  - http://intrologic.stanford.edu/exercises/exercise\_03\_04.html

#### Exercise 3.4.3

#### If $\Gamma \vDash \phi$ and $\Delta \nvDash \phi$ , then $\Gamma \cup \Delta \vDash \phi$

For the second question  $\Gamma \vDash \varphi$  and  $\Delta \nvDash \varphi$ , then  $\Gamma \cup \Delta \vDash \varphi$ . WLOG, this time let  $\Gamma = \{a,b\}$  and  $\Delta = \{b,c\}$ .  $\Gamma \vDash \varphi$  means any truth assignment that satisfies a **and** b also satisfy  $\varphi$ , i.e.  $a \land b \Rightarrow \varphi$ .  $\Delta \nvDash \varphi$  means **some** truth assignment that satisfies b and c does not satisfy  $\varphi$ . Now  $\Gamma \cup \Delta = \{a,b,c\}$ , so the question is about whether  $a \land b \land c \Rightarrow \varphi$ . Notice that any truth assignment that satisfies  $a \land b \land c$  **must** satisfy  $a \land b$  as well, so it also satisfies  $\varphi$ , and we have that  $\Gamma \cup \Delta$  entails  $\varphi$ . The fact that  $\Delta \nvDash \varphi$  has nothing to do with it, since we know  $\varphi$  must be true as long as  $a \land b$  is true.

IMO, you just need to be aware that  $\Gamma \cup \Delta \neq \Gamma \vee \Delta$  - this seems to be an easy mistake.

# Logical Equivalence

- $\phi$  and  $\Psi$  are logically equivalent if they entail each other
- Equivalence Theorem:
  - $\phi$  and  $\Psi$  are equivalent if ( $\phi \Leftrightarrow \Psi$ )
- Example 1:
  - $((p \Rightarrow q) \lor (q \Rightarrow r))$  and  $(p \lor \neg p)$
- Example 2:
  - $(p \land q \Rightarrow r)$  and  $(p \land r \Rightarrow q)$
- Extra Practice: <u>http://intrologic.stanford.edu/exercises/exercise\_03\_02.html</u>

# Logical Consistency

- $\phi$  and  $\Psi$  are consistent if there is a truth assignment that satisfies both
- Example 1:
  - $\{p \Rightarrow r, q \Rightarrow r, p \lor q\}$  and r
- Example 2:
  - $\{p \Rightarrow r, q \Rightarrow r, p \lor q\}$  and  $\neg r$
- Extra Practice: http://intrologic.stanford.edu/exercises/exercise\_03\_05.html

# Soundness & Completeness

# Soundness and Completeness

- Soundness: a proof system is sound iff every conclusion that is provable from a set of premises is logically entailed.
  - $\Delta \vdash \phi$ , then  $\Delta \vDash \phi$
  - Everything derivable / provable is true
- Completeness: a proof system is complete iff every conclusion that is logically entailed by a set of premises is provable
  - $\Delta \vDash \varphi$ , then  $\Delta \upharpoonright \varphi$
  - Everything true is derivable / provable

Hilbert? Yes and Yes Fitch? Yes and Yes

# **Hilbert Proofs**

### **Hilbert Proofs**

Implication Elimination

 $\begin{aligned} \varphi \Rightarrow \psi \\ \varphi \\ \hline \psi \\ \hline \end{aligned}$ 

Implication Creation (IC) $\phi \Rightarrow (\psi \Rightarrow \phi)$ Implication Distribution (ID) $(\phi \Rightarrow (\psi \Rightarrow \chi)) \Rightarrow ((\phi \Rightarrow \psi) \Rightarrow (\phi \Rightarrow \chi))$ Implication Reversal (IR) $(\neg \psi \Rightarrow \neg \phi) \Rightarrow (\phi \Rightarrow \psi)$ 

### Live Demo. Premises: p and ~p Prove: q

http://intrologic.stanford.edu/logica/ homepage/hilbert.php



#### Introduction to Logic

Tools for Thought

	Hilbert									
		Undo	Сору	Paste	Load	Save	Help			
	Select Al	11								
□1.	р					Prem	ise			
□2.	~p					Prem	ise			
□3.	~p => (~	q => ~p)				Impli	cation Creation			
	~q => ~p					Impli	cation Elimination: 3, 2			
□5.	(~q => ~	p) => (p =	<b>&gt;</b> q)			Impli	cation Reversal			
$\Box 6. p \Rightarrow q$				Implication Elimination: 5, 4						
□7.	q					Impli	cation Elimination: 6, 1			
Goal	q					Con	nplete			
		Pren	nise	Implicatio	n Creation	Implicatio	n Elimination			
		Reiter	ation	Implication	Distribution	Universal	Generalizatior			
		Truth	table	Implicatio	n Reversal	Domai	n Closure			
		Shor	tcut	Universal	Specializatio	n Ind	luction			
Replace				Universal	Distribution					
		Coale								
		Del	ete	J						
Show Answer Reset										

## Live Demo. Premises: ~q and ~p => (~q=>~r) Prove: r=>p

http://intrologic.stanford.edu/logica/ homepage/hilbert.php



#### Introduction to Logic

Profile Sign Out

Practice Test - Problem 3						
Undo <b>F</b>	lelp					
Select All						
□1. ~q	Premise					
$\Box 2.  \sim p \Longrightarrow (\sim q \Longrightarrow \sim r)$	Premise					
$\Box$ 3. r => p	Goal					
$\Box 4. \ (\sim p \Longrightarrow (\sim q \Longrightarrow \sim r)) \Longrightarrow ((\sim p \Longrightarrow \sim q) \Longrightarrow (\sim p \Longrightarrow \sim q) \Longrightarrow (\sim p \Longrightarrow \sim q)$	r)) Implication Distribution					
$\Box 5. (~p \Rightarrow ~q) \Rightarrow (~p \Rightarrow ~r)$	Implication Elimination: 4, 2					
$\Box 6.  \sim q \Longrightarrow (\sim p \Longrightarrow \sim q)$	Implication Creation					
$\Box$ 7. ~p => ~q	Implication Elimination: 6, 1					
$\Box 8. \sim p \Longrightarrow \sim r$	Implication Elimination: 5, 7					
$\Box 9.  (\sim p \Longrightarrow \sim r) \Longrightarrow (r \Longrightarrow p)$	Implication Reversal					
□ 10. r ⇒ p	Implication Elimination: 9,8					
Goal	Incomplete					
Premise Implication Cre	ation Implication Elimination					
Reiteration Implication Distr						
Truthtable Implication Rev						
Shortcut Universal Specia						
Replace Universal Distri	bution					
Coalesce						
Delete						

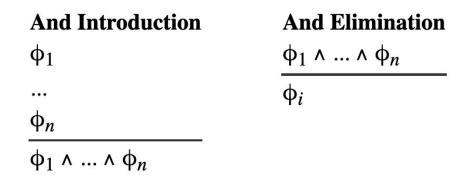
Submit	Reset
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# Dominic's Qn

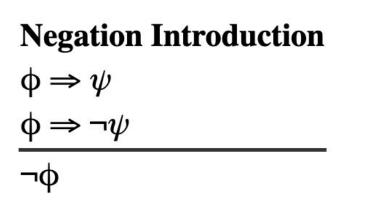
	Select All	
□1.	$p \ll p$	Premise
□2.	$p \Rightarrow p$	Biconditional Elimination: 1
□ 3.	$\sim p \Longrightarrow p$	Biconditional Elimination: 1
□4.	р	Assumption
□ 5.	~p	Implication Elimination: 2, 4
□ 6.	р	Implication Elimination: 3, 5
□ 7.	p => p	Implication Introduction: 4, 6
□ 8.	~p	Negation Introduction: 7, 2
□9.	~p	Assumption
□ <b>10</b> .	р	Implication Elimination: 3, 9
□ 11.	~p	Implication Elimination: 2, 10
□ 12.	$\sim p \Longrightarrow \sim p$	Implication Introduction: 9, 11
□ 13.	~~p	Negation Introduction: 3, 12
□ 14.	p	Negation Elimination: 13
□ 15.	$\sim$ q	Assumption
□ 16.	р	Reiteration: 14
□ 17.	$\sim q \Rightarrow p$	Implication Introduction: 15, 16
□ 18.	~q	Assumption
□ 19.	~p	Reiteration: 8
□ 20.	$\sim q \Rightarrow \sim p$	Implication Introduction: 18, 19
□ 21.	~~q	Negation Introduction: 17, 20
□ 22.	q	Negation Elimination: 21

 $\rightarrow$  Several comments on Ed / in OH about how Fitch Proofs seem unintuitive / unnatural (especially when making assumptions)

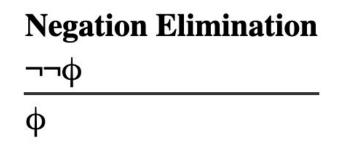
- This is normal, and you will get better with practice!
- Resources: <u>Lesson Exercises</u> (especially 5.11-5.14, some of which we will go through), Practice Quiz, <u>Past Quizzes</u>
- $\rightarrow$  Some things to note
  - Don't just focus on the premises (what you have to work with), take note of the goal (where you're trying to go)
    - This will help with 1. choice of assumptions / subproofs and 2. choice of rules
  - Be familiar with your toolbox i.e. the rules you can use
  - Mindset: given the info that I have (premises) and the tools I have to work with (rules), how do I get to the goal

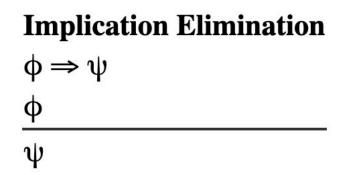


<b>Or Introduction</b>	<b>Or Elimination</b>
$\phi_i$	$\phi_1 \vee \vee \phi_n$
$\overline{\phi_1 \vee \vee \phi_n}$	$\varphi_1 \Rightarrow \psi$
	$\phi_n \Rightarrow \psi$



<b>Implication Introduction</b>
$\phi \vdash \psi$
$\varphi \Rightarrow \psi$





# **Biconditional Introduction**

 $\begin{aligned} \varphi \Rightarrow \psi \\ \psi \Rightarrow \varphi \\ \hline \varphi \Leftrightarrow \psi \end{aligned}$ 

Biconditional Elimination  $\phi \Leftrightarrow \psi$   $\phi \Rightarrow \psi$  $\psi \Rightarrow \phi$ 

## Other things to note about the interface

- $\rightarrow$  Premise operation: Allows one to add a new premise to a proof
- $\rightarrow$  Reiteration operation: Allows one to reproduce an earlier conclusion for the purposes of clarity.
- $\rightarrow$  Delete operation: Allows one to delete unnecessary lines.
- $\rightarrow$  **Reasoning Tips**: Lecture 5 (Natural Deduction) Slides 39-42

# Live Demo Fitch

## Exercise 5.11

	Fitch							
	Undo	Сору	Paste	Load	Save	Help		
2	Select All							
⊐1.	p => q				Premise			
⊐2.	~(~p   q)				Assumptio	on		
⊐3.	р				Assumptio	n		
⊐4.	q				Implicatio	n Elimination: 1, 3		
⊐5.	~p q				Or Introdu	ction: 4		
□6.	p => ~p   o	q			Implicatio	n Introduction: 3, 5		
⊐7.	р				Assumptio	n		
⊐8.	~(~p   q	)			Reiteration	n: 2		
⊐9.	p => ~(~p	(plq			Implicatio	n Introduction: 7, 8		
□ 10.	~p				Negation I	introduction: 6,9		
⊐11.	~(~p   q) =>	~p			Implicatio	n Introduction: 2, 10		
⊐12.	~(~p   q)				Assumptio	on		
⊐13.	~p				Assumptio	on		
□14.	~p q				Or Introdu	ction: 13		
⊐15.	~p => ~p	l q			Implicatio	n Introduction: 13, 14		
□16.	~p				Assumptio	on		
⊐17.	~(~p   q	)			Reiteration	n: 12		
□18.	~p => ~(~	p q)			Implicatio	n Introduction: 16, 17		
⊐19.	~~p				Negation I	introduction: 15, 18		
⊇20.	~(~p   q) =>	~~p			Implicatio	n Introduction: 12, 19		
⊐21.	~~(~p   q)				Negation I	introduction: 11, 20		
⊇22.	~p   q				Negation I	Elimination: 21		
	Sett 10054				80C17.			
Goal	~p   q				Co	mplete		

# Ed Post

				Fitch			
	Undo	Сору	Paste	Load	Save	Library	Help
+ •	Objects:						
+ -	Functions:						
	Select All						
□1.	s & t   ~s & ~	٠t				Premise	
⊇2.	s & t					Assumptio	on
□3.	s					Assumptio	on
⊇4.	t					And Elimi	ination: 2
□5.	s => t					Implicatio	on Introduction: 3, 4
□6.	s & t => (s =:	> t)				Implicatio	on Introduction: 2, 5
□7.	~s & ~t					Assumptio	on
□8.	s					Assumptio	on
⊒9.	~t					Assumptio	on
□10.	s					Reiteration	n: 8
□11.	~t => s					Implicatio	n Introduction: 9, 10
□12.	~t					Assumptio	on
□13.	~s					And Elimi	ination: 7
□14.	~t => ~s					Implicatio	n Introduction: 12, 13
□15.	~~t					Negation 1	Introduction: 11, 14
□16.	t					Negation I	Elimination: 15
□17.	s => t					Implicatio	on Introduction: 8, 16
□18.	~s & ~t => (s	s => t)				Implicatio	on Introduction: 7, 17
□19.	s => t					Or Elimin	ation: 1, 6, 18

### 2021 Quiz 1 Problem 4 (requested on Ed)

Problem 4 - Fitch							
	Undo Help						
	Select All						
□1.	~q		Premise				
□2.	${\sim}p \Longrightarrow ({\sim}q \Longrightarrow {\sim}r)$		Premise				
□3.	slr		Premise				
□4.	s => t		Premise				
□5.	p => t		Premise				
□6.	r		Assumption				
□7.	~p		Assumption				
□8.	r		Reiteration: 6				
□9.	~p => r		Implication Introduction: 7, 8				
□10.	~p		Assumption				
□11.	~q => ~r		Implication Elimination: 2, 10				
□12.	~r		Implication Elimination: 11, 1				
□13.	~p => ~r		Implication Introduction: 10, 12				
□14.	~~p		Negation Introduction: 9, 13				
□15.	р		Negation Elimination: 14				
□16.	t		Implication Elimination: 5, 15				
□17.	r => t		Implication Introduction: 6, 16				
□18.	t		Or Elimination: 3, 4, 17				
Goal	t		Complete				

# **Resolution Proofs**

# Clausal Form - INDO (I)

Implications:

$$\begin{array}{lll} \varphi \Rightarrow \psi & \rightarrow & \neg \varphi \lor \psi \\ \varphi \Leftarrow \psi & \rightarrow & \varphi \lor \neg \psi \\ \varphi \Leftrightarrow \psi & \rightarrow & (\neg \varphi \lor \psi) \land (\varphi \lor \neg \psi) \end{array}$$

# Clausal Form - INDO (N)

Negation:

$$\begin{array}{lll} \neg \varphi & \rightarrow & \varphi \\ \neg (\varphi \land \psi) & \rightarrow & \neg \varphi \lor \neg \psi \\ \neg (\varphi \lor \psi) & \rightarrow & \neg \varphi \land \neg \psi \end{array}$$

# Clausal Form - INDO (I)

Distribution:

# Clausal Form - INDO (I)

Operators:

# Clausal Form Example

$$\neg(g \land (r \Rightarrow f))$$

# **Resolution Principle**

$$\begin{cases} \varphi_1, \dots, \chi, \dots, \varphi_m \\ \frac{\{\psi_1, \dots, \neg \chi, \dots, \psi_n\}}{\{\psi_1, \dots, \neg \chi, \dots, \psi_n\}} \\ \\ \hline \{\varphi, q\} \\ \frac{\{p, q\}}{\{q\}} \\ \hline \{p, r\} \\ \hline \{q, \neg q\} \\ \hline$$

### **Resolution Proof**

http://intrologic.stanford.edu/exercises/exercise\_06\_04.html



Tools for Thought

Exercise 6.4 - Resolution

Given the premises  $(p \Rightarrow q)$  and  $(r \Rightarrow s)$ , use Propositional Resolution to prove the conclusion  $(p \lor r \Rightarrow q \lor s)$ .

Show Instructions

	R	esolution					
Sel	ect All						
□ 1. {~ <b>i</b>	p,q}	Premise					
□ 2. {~ <b>1</b>	;,s}	Premise					
□ 3. {p,	r}	Goal					
□ 4. {~0	1}	Goal					
□ 5. {~s	5}	Goal					
□ 6. {~ <u>1</u>	p}	Resolution: 1, 4					
□ 7. {~r	·}	Resolution: 2, 5					
□ 8. {r}		Resolution: 3, 6					
□ <b>9</b> . {}		Resolution: 8, 7					
Goal {}		Complete					
	Premise	Resolution					
	Goal						
	Reiteration Delete						
Show Answer Reset Show XML							

# **Resolution Proof**

http://intrologic.stanford.edu/stanford/problem.php?test=practice&problem=proble m5



#### Introduction to Logic

Profile Sign Out

Practice Test - Problem 5									
NB: We will sav	NB: We will save and grade only the first 30 lines of your proof. Be economical.								
	Undo	Help							
Select All									
$\Box$ 1. {p,~r}			Premise						
$\Box$ 2. {~p,r}			Premise						
□ 3. {~q,~r}			Premise						
□ 4. {q,r}			Premise						
□ 5. {~q,~s}			Premise						
$\Box$ 6. {s,p}			Premise						
□ 7. {~p,~r,q}			Premise						
□ 8. {p,~q}			Resolution: 6, 5						
□ 9. {~q,r}			Resolution: 8, 2						
□ 10. {r}			Resolution: 4, 9						
□ 11. {p}			Resolution: 10, 1						
□ 12. {~q}			Resolution: 10, 3						
□ 13. {~p,~r}			Resolution: 7, 12						
□ 14. {~p}			Resolution: 10, 13						
□ 15. {}			Resolution: 11, 14						
Goal {}			Complete						
	Goal	Resolution							
	Reiteration	Factor	]						
	Delete								

# Q&A