CS 157 Quiz 1 Review

October 17th 2023

Agenda

1. Validity, Unsatisfiability, Contingency
2. Logical Entailment, Equivalence, Consistency
3. Soundness and Completeness
4. Hilbert Proofs
5. Natural Deduction & Fitch Proofs
6. Resolution

Validity, Unsatisfiability, Contingency

Propositional Logic & Analysis

→ **Valid**: Satisfied by *every truth assignment*.

- E.g. $(p \lor \neg p)$.
- Always yields T in a truth table

→ **Unsatisfiable**: Not satisfied by *any* truth assignment.

- E.g. $(p \land \neg p)$
- Always yields F in a truth table

→ **Contingent:** There is some truth assignment that satisfies it, and some truth assignment that falsifies it

- E.g. $(p \land q)$: If p and q both true, it is true. If p and q both false, it is false

Logical Entailment, Equivalence, Consistency

Logical Entailment

- A set of sentences logically entails a sentence iff every truth assignment that satisfies the premises also satisfies the conclusion
- Example 1:
  - $\{p \Rightarrow r\} \models (p \Rightarrow q \lor r)$
- Example 2:
  - $\{p \Rightarrow q \lor r, p \Rightarrow r\} \models (q \Rightarrow r)$
- Extra Practice:
  - http://intrologic.stanford.edu/exercises/exercise_03_03.html
  - http://intrologic.stanford.edu/exercises/exercise_03_04.html

Exercise 3.4.3

If $\Gamma \vDash \varphi$ and $\Delta \nvDash \varphi$, then $\Gamma \cup \Delta \vDash \varphi$

For the second question $\Gamma \vDash \varphi$ and $\Delta \nvDash \varphi$, then $\Gamma \cup \Delta \vDash \varphi$. WLOG, this time let $\Gamma = \{a,b\}$ and $\Delta = \{b,c\}$. $\Gamma \vDash \varphi$ means any truth assignment that satisfies a **and** b also satisfy φ, i.e. $a \wedge b \Rightarrow \varphi$. $\Delta \nvDash \varphi$ means **some** truth assignment that satisfies b and c does not satisfy φ. Now $\Gamma \cup \Delta = \{a,b,c\}$, so the question is about whether $a \wedge b \wedge c \Rightarrow \varphi$. Notice that any truth assignment that satisfies $a \wedge b \wedge c$ **must** satisfy $a \wedge b$ as well, so it also satisfies φ, and we have that $\Gamma \cup \Delta$ entails φ. The fact that $\Delta \nvDash \varphi$ has nothing to do with it, since we know φ must be true as long as $a \wedge b$ is true.

IMO, you just need to be aware that $\Gamma \cup \Delta \neq \Gamma \vee \Delta$ - this seems to be an easy mistake.

Logical Equivalence

- ϕ and Ψ are logically equivalent if they entail each other
- Equivalence Theorem:
  - ϕ and Ψ are equivalent if $(\phi \Leftrightarrow \Psi)$
- Example 1:
  - $((p \Rightarrow q) \vee (q \Rightarrow r))$ and $(p \vee \neg p)$
- Example 2:
  - $(p \wedge q \Rightarrow r)$ and $(p \wedge r \Rightarrow q)$
- Extra Practice: http://intrologic.stanford.edu/exercises/exercise_03_02.html

Logical Consistency

- ϕ and Ψ are consistent if there is a truth assignment that satisfies both
- Example 1:
  - $\{p \Rightarrow r, q \Rightarrow r, p \vee q\}$ and r
- Example 2:
  - $\{p \Rightarrow r, q \Rightarrow r, p \vee q\}$ and $\neg r$
- Extra Practice: http://intrologic.stanford.edu/exercises/exercise_03_05.html

Soundness & Completeness

Soundness and Completeness

- Soundness: a proof system is sound iff every conclusion that is provable from a set of premises is logically entailed.
  - $\Delta \vdash \varphi$, then $\Delta \vDash \varphi$
  - Everything derivable / provable is true
- Completeness: a proof system is complete iff every conclusion that is logically entailed by a set of premises is provable
  - $\Delta \vDash \varphi$, then $\Delta \vdash \varphi$
  - Everything true is derivable / provable

Hilbert? Yes and Yes
Fitch? Yes and Yes

Hilbert Proofs

Hilbert Proofs

Implication Elimination

$$\frac{\begin{array}{l}\phi \Rightarrow \psi \\ \phi\end{array}}{\psi}$$

Implication Creation (IC)	$\phi \Rightarrow (\psi \Rightarrow \phi)$
Implication Distribution (ID)	$(\phi \Rightarrow (\psi \Rightarrow \chi)) \Rightarrow ((\phi \Rightarrow \psi) \Rightarrow (\phi \Rightarrow \chi))$
Implication Reversal (IR)	$(\neg\psi \Rightarrow \neg\phi) \Rightarrow (\phi \Rightarrow \psi)$

Live Demo. Premises: p and ~p Prove: q

http://intrologic.stanford.edu/logica/homepage/hilbert.php

Introduction to Logic

Tools for Thought

Hilbert

Undo Copy Paste Load Save Help

	Select All	
1.	p	Premise
2.	~p	Premise
3.	~p => (~q => ~p)	Implication Creation
4.	~q => ~p	Implication Elimination: 3, 2
5.	(~q => ~p) => (p => q)	Implication Reversal
6.	p => q	Implication Elimination: 5, 4
7.	q	Implication Elimination: 6, 1

Goal q Complete

Premise | Implication Creation | Implication Elimination
Reiteration | Implication Distribution | Universal Generalizatio
Truthtable | Implication Reversal | Domain Closure
Shortcut | Universal Specialization | Induction
Replace | Universal Distribution
Coalesce
Delete

Show Answer Reset

Live Demo. Premises: ~q and ~p => (~q=>~r) Prove: r=>p

http://intrologic.stanford.edu/logica/homepage/hilbert.php

Introduction to Logic

Profile
Sign Out

Practice Test - Problem 3

Undo Help

☐ Select All

1.	~q	Premise
2.	~p => (~q => ~r)	Premise
3.	r => p	Goal
4.	(~p => (~q => ~r)) => ((~p => ~q) => (~p => ~r))	Implication Distribution
5.	(~p => ~q) => (~p => ~r)	Implication Elimination: 4, 2
6.	~q => (~p => ~q)	Implication Creation
7.	~p => ~q	Implication Elimination: 6, 1
8.	~p => ~r	Implication Elimination: 5, 7
9.	(~p => ~r) => (r => p)	Implication Reversal
10.	r => p	Implication Elimination: 9, 8

Goal — Incomplete

Premise	Implication Creation	Implication Elimination
Reiteration	Implication Distribution	Universal Generalization
Truthtable	Implication Reversal	Domain Closure
Shortcut	Universal Specialization	Induction
Replace	Universal Distribution	
Coalesce		
Delete		

Submit Reset

Dominic's Qn

☐	Select All	
☐ 1.	p <=> ~p	Premise
☐ 2.	p => ~p	Biconditional Elimination: 1
☐ 3.	~p => p	Biconditional Elimination: 1
☐ 4.	│ p	Assumption
☐ 5.	│ ~p	Implication Elimination: 2, 4
☐ 6.	│ p	Implication Elimination: 3, 5
☐ 7.	p => p	Implication Introduction: 4, 6
☐ 8.	~p	Negation Introduction: 7, 2
☐ 9.	│ ~p	Assumption
☐ 10.	│ p	Implication Elimination: 3, 9
☐ 11.	│ ~p	Implication Elimination: 2, 10
☐ 12.	~p => ~p	Implication Introduction: 9, 11
☐ 13.	~~p	Negation Introduction: 3, 12
☐ 14.	p	Negation Elimination: 13
☐ 15.	│ ~q	Assumption
☐ 16.	│ p	Reiteration: 14
☐ 17.	~q => p	Implication Introduction: 15, 16
☐ 18.	│ ~q	Assumption
☐ 19.	│ ~p	Reiteration: 8
☐ 20.	~q => ~p	Implication Introduction: 18, 19
☐ 21.	~~q	Negation Introduction: 17, 20
☐ 22.	q	Negation Elimination: 21

Natural Deduction & Fitch Proofs

Natural Deduction: Fitch Proofs

→ Several comments on Ed / in OH about how Fitch Proofs seem unintuitive / unnatural (especially when making assumptions)

- This is normal, and you will get better with practice!
- Resources: Lesson Exercises (especially 5.11-5.14, some of which we will go through), Practice Quiz, Past Quizzes

→ Some things to note

- Don't just focus on the premises (what you have to work with), take note of the goal (where you're trying to go)
    - This will help with 1. choice of assumptions / subproofs and 2. choice of rules
- Be familiar with your toolbox i.e. the rules you can use
- Mindset: given the info that I have (premises) and the tools I have to work with (rules), how do I get to the goal

Natural Deduction: Fitch Proofs

And Introduction

ϕ_1

...

ϕ_n

$\phi_1 \wedge \ldots \wedge \phi_n$

And Elimination

$\phi_1 \wedge \ldots \wedge \phi_n$

ϕ_i

Or Introduction

ϕ_i

$\phi_1 \vee \ldots \vee \phi_n$

Or Elimination

$\phi_1 \vee \ldots \vee \phi_n$

$\phi_1 \Rightarrow \psi$

...

$\phi_n \Rightarrow \psi$

ψ

Natural Deduction: Fitch Proofs

Negation Introduction

$\phi \Rightarrow \psi$

$\phi \Rightarrow \neg\psi$

$\neg\phi$

Negation Elimination

$\neg\neg\phi$

ϕ

Implication Introduction

$\phi \vdash \psi$

$\phi \Rightarrow \psi$

Implication Elimination

$\phi \Rightarrow \psi$

ϕ

ψ

Natural Deduction: Fitch Proofs

Biconditional Introduction

$\phi \Rightarrow \psi$

$\psi \Rightarrow \phi$

$\phi \Leftrightarrow \psi$

Biconditional Elimination

$\phi \Leftrightarrow \psi$

$\phi \Rightarrow \psi$

$\psi \Rightarrow \phi$

Other things to note about the interface

→ Premise operation: Allows one to add a new premise to a proof

→ Reiteration operation: Allows one to reproduce an earlier conclusion for the purposes of clarity.

→ Delete operation: Allows one to delete unnecessary lines.

→ **Reasoning Tips**: Lecture 5 (Natural Deduction) Slides 39-42

Live Demo

Fitch

Exercise 5.11

Fitch

Undo Copy Paste Load Save Help

Select All

1.	p => q	Premise
2.	~(~p | q)	Assumption
3.	p	Assumption
4.	q	Implication Elimination: 1, 3
5.	~p | q	Or Introduction: 4
6.	p => ~p | q	Implication Introduction: 3, 5
7.	p	Assumption
8.	~(~p | q)	Reiteration: 2
9.	p => ~(~p | q)	Implication Introduction: 7, 8
10.	~p	Negation Introduction: 6, 9
11.	~(~p | q) => ~p	Implication Introduction: 2, 10
12.	~(~p | q)	Assumption
13.	~p	Assumption
14.	~p | q	Or Introduction: 13
15.	~p => ~p | q	Implication Introduction: 13, 14
16.	~p	Assumption
17.	~(~p | q)	Reiteration: 12
18.	~p => ~(~p | q)	Implication Introduction: 16, 17
19.	~~p	Negation Introduction: 15, 18
20.	~(~p | q) => ~~p	Implication Introduction: 12, 19
21.	~~(~p | q)	Negation Introduction: 11, 20
22.	~p | q	Negation Elimination: 21
Goal	~p | q	Complete

Ed Post

Fitch

Undo Copy Paste Load Save Library Help

Objects:

Functions:

Select All

1.	s & t | ~s & ~t	Premise
2.	s & t	Assumption
3.	s	Assumption
4.	t	And Elimination: 2
5.	s => t	Implication Introduction: 3, 4
6.	s & t => (s => t)	Implication Introduction: 2, 5
7.	~s & ~t	Assumption
8.	s	Assumption
9.	~t	Assumption
10.	s	Reiteration: 8
11.	~t => s	Implication Introduction: 9, 10
12.	~t	Assumption
13.	~s	And Elimination: 7
14.	~t => ~s	Implication Introduction: 12, 13
15.	~~t	Negation Introduction: 11, 14
16.	t	Negation Elimination: 15
17.	s => t	Implication Introduction: 8, 16
18.	~s & ~t => (s => t)	Implication Introduction: 7, 17
19.	s => t	Or Elimination: 1, 6, 18

2021 Quiz 1 Problem 4 (requested on Ed)

Problem 4 - Fitch

Undo Help

	Select All	
1.	~q	Premise
2.	~p => (~q => ~r)	Premise
3.	s | r	Premise
4.	s => t	Premise
5.	p => t	Premise
6.	r	Assumption
7.	~p	Assumption
8.	r	Reiteration: 6
9.	~p => r	Implication Introduction: 7, 8
10.	~p	Assumption
11.	~q => ~r	Implication Elimination: 2, 10
12.	~r	Implication Elimination: 11, 1
13.	~p => ~r	Implication Introduction: 10, 12
14.	~~p	Negation Introduction: 9, 13
15.	p	Negation Elimination: 14
16.	t	Implication Elimination: 5, 15
17.	r => t	Implication Introduction: 6, 16
18.	t	Or Elimination: 3, 4, 17
Goal	t	Complete

Resolution Proofs

Clausal Form - INDO (I)

Implications:

$$\phi \Rightarrow \psi \quad \rightarrow \quad \neg\phi \vee \psi$$

$$\phi \Leftarrow \psi \quad \rightarrow \quad \phi \vee \neg\psi$$

$$\phi \Leftrightarrow \psi \quad \rightarrow \quad (\neg\phi \vee \psi) \wedge (\phi \vee \neg\psi)$$

Clausal Form - INDO (N)

Negation:

$$\neg\neg\phi \rightarrow \phi$$

$$\neg(\phi \wedge \psi) \rightarrow \neg\phi \vee \neg\psi$$

$$\neg(\phi \vee \psi) \rightarrow \neg\phi \wedge \neg\psi$$

Clausal Form - INDO (I)

Distribution:

$$
\begin{array}{lcl}
\phi \vee (\psi \wedge \chi) & \rightarrow & (\phi \vee \psi) \wedge (\phi \vee \chi) \\
(\phi \wedge \psi) \vee \chi & \rightarrow & (\phi \vee \chi) \wedge (\psi \vee \chi) \\
\phi \vee (\phi_1 \vee \ldots \vee \phi_n) & \rightarrow & \phi \vee \phi_1 \vee \ldots \vee \phi_n \\
(\phi_1 \vee \ldots \vee \phi_n) \vee \phi & \rightarrow & \phi_1 \vee \ldots \vee \phi_n \vee \phi \\
\phi \wedge (\phi_1 \wedge \ldots \wedge \phi_n) & \rightarrow & \phi \wedge \phi_1 \wedge \ldots \wedge \phi_n \\
(\phi_1 \wedge \ldots \wedge \phi_n) \wedge \phi & \rightarrow & \phi_1 \wedge \ldots \wedge \phi_n \wedge \phi
\end{array}
$$

Clausal Form - INDO (I)

Operators:

$$\phi_1 \vee \ldots \vee \phi_n \quad \rightarrow \quad \{\phi_1, \ldots, \phi_n\}$$

$$\phi_1 \wedge \ldots \wedge \phi_n \quad \rightarrow \quad \{\phi_1\}, \ldots, \{\phi_n\}$$

Clausal Form Example

$$\neg(g \land (r \Rightarrow f))$$

Resolution Principle

$$\frac{\{\phi_1, \dots, \chi, \dots, \phi_m\} \quad \{\psi_1, \dots, \neg\chi, \dots, \psi_n\}}{\{\phi_1, \dots, \phi_m, \psi_1, \dots, \psi_n\}}$$

$$\frac{\{\neg p, q\} \quad \{p, q\}}{\{q\}}$$

$$\frac{\{p, q\} \quad \{\neg q, r\}}{\{p, r\}}$$

$$\frac{\{p\} \quad \{\neg p\}}{\{\}}$$

$$\frac{\{p, q\} \quad \{\neg p, \neg q\}}{\{p, \neg p\} \quad \{q, \neg q\}}$$

Resolution Proof

http://intrologic.stanford.edu/exercises/exercise_06_04.html

Introduction to Logic

Tools for Thought

Exercise 6.4 - Resolution

Given the premises ($p \Rightarrow q$) and ($r \Rightarrow s$), use Propositional Resolution to prove the conclusion ($p \vee r \Rightarrow q \vee s$).

Show Instructions

Resolution

☐		Select All	
☐	1.	{~p,q}	Premise
☐	2.	{~r,s}	Premise
☐	3.	{p,r}	Goal
☐	4.	{~q}	Goal
☐	5.	{~s}	Goal
☐	6.	{~p}	Resolution: 1, 4
☐	7.	{~r}	Resolution: 2, 5
☐	8.	{r}	Resolution: 3, 6
☐	9.	{}	Resolution: 8, 7

Goal	{}	Complete

Premise | Resolution

Goal

Reiteration

Delete

Show Answer | Reset | Show XML

Resolution Proof

http://intrologic.stanford.edu/stanford/problem.php?test=practice&problem=problem5

Practice Test - Problem 5

NB: We will save and grade only the first 30 lines of your proof. Be economical.

Undo Help

Select All

1.	{p,~r}	Premise
2.	{~p,r}	Premise
3.	{~q,~r}	Premise
4.	{q,r}	Premise
5.	{~q,~s}	Premise
6.	{s,p}	Premise
7.	{~p,~r,q}	Premise
8.	{p,~q}	Resolution: 6, 5
9.	{~q,r}	Resolution: 8, 2
10.	{r}	Resolution: 4, 9
11.	{p}	Resolution: 10, 1
12.	{~q}	Resolution: 10, 3
13.	{~p,~r}	Resolution: 7, 12
14.	{~p}	Resolution: 10, 13
15.	{}	Resolution: 11, 14

Goal {} Complete

Premise Resolution
Goal Factor
Reiteration
Delete

Q&A