CS 157: Midterm Examination 2
Fall 2019-20

- Please read all instructions (including these) carefully.
- The exam is closed book, closed notes, and closed internet.
- The exam consists of 9 pages including this page. The last page is a reference sheet for the Fitch rules of inference. There are 5 questions. Each question is worth 10 points.
- Time limit: 1 hour. Budget your time accordingly.
- Please write your solutions in the spaces provided on the exam. Make sure that your solutions are neat and clearly marked. You may use the blank areas and backs of the exam pages for scratch work.

In accordance with both the letter and spirit of the Honor Code, I have neither given nor received assistance on this examination.

NAME: 

SUNETID (username): 

SIGNATURE: 

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Question 1 [10 points]

Please circle exactly one of the provided answers (no explanation is required).

(a) Is the following sentence valid, contingent, or unsatisfiable? (5 points)

\[(p \iff \neg q) \Rightarrow (\neg p \vee \neg q)\]

VALID \hspace{1cm} CONTINGENT \hspace{1cm} UNSATISFIABLE

(b) Is the following sentence valid, contingent, or unsatisfiable? (5 points)

\[((p \land \neg q) \Rightarrow (q \land r)) \iff (p \land \neg q)\]

VALID \hspace{1cm} CONTINGENT \hspace{1cm} UNSATISFIABLE
Question 2 [10 points]

Please circle exactly one of the provided answers (no explanation is required).

(a) Fitch is complete for Propositional Logic; that is, for a set of clauses $\Delta$ where $\Delta \vDash \phi$, then there is guaranteed to be a Fitch proof of $\phi$ from $\Delta$.

TRUE   FALSE

(b) Resolution is complete for Propositional Logic; that is, for a set of clauses $\Delta$ with $\Delta \vDash \phi$, then there is guaranteed to be a Resolution proof of $\phi$ from $\Delta$.

TRUE   FALSE

(c) Consider a proof system with the single rule of inference $\{\} \vdash \phi$ for all $\phi$. Is this proof system sound, complete, both, or neither?

SOUND   COMPLETE   BOTH   NEITHER
Question 3 [10 points]

Assume that $\Gamma$ and $\Delta$ are sets of sentences in Propositional Logic, and $\phi$ and $\psi$ are individual sentences in Propositional Logic. Please circle true or false for each of the following statements (no explanation is required).

(a) If $\Gamma \not\models \phi$ and $\Gamma \not\models \psi$, then $\Gamma \not\models \{\phi, \psi\}$.

TRUE  FALSE

(b) If $\Gamma \models \phi$ and $\Delta \models \phi$, then $\Gamma \cap \Delta \models \phi$.

TRUE  FALSE

(c) If $\Gamma \not\models \phi$ and $\Delta \not\models \phi$, then $\Gamma \cup \Delta \not\models \phi$.

TRUE  FALSE

(d) If $\Gamma \models \phi$ and $\Gamma \not\models \neg \phi$, then $\Gamma$ is consistent.

TRUE  FALSE

(e) If $\Gamma \models \phi$ and $\Delta \models \neg \phi$, then $\phi$ is contingent.

TRUE  FALSE
Question 4 [10 points]

Given the premise $\neg p \rightarrow r$, prove $p \lor r$ using the Fitch System.

Please annotate your proof by writing the rule of inference and line number(s) for each step in your proof (abbreviations are fine). Clearly mark any assumptions and subproofs within your proof in the same format as the exercises.

1. $\neg p \rightarrow r$  
   Premise
Question 5 [10 points]

(a) Convert the following sentence to clausal form. (4 points)

\[
(\neg p \rightarrow \neg (q \vee r)) \rightarrow (p \land \neg r)
\]

*Hint:* Our conversion yields 4 clauses.
(b) Prove \( \neg q \land \neg r \) using Propositional Resolution given the premises supplied below:

\[
(q \rightarrow p) \land (r \rightarrow q) \\
\neg((p \land r) \lor \neg(q \rightarrow r))
\]
Fitch Rules of Inference and Induction Axioms

And Introduction

\[ \phi_1 \]
\[ \vdots \]
\[ \phi_n \]

\[ \phi_1 \land \cdots \land \phi_n \]

And Elimination

\[ \phi_1 \land \cdots \land \phi_n \]

\[ \phi_i \]

Or Introduction

\[ \phi_1 \]

\[ \phi_1 \lor \cdots \lor \phi_n \]

Or Elimination

\[ \phi_1 \lor \cdots \lor \phi_n \]

\[ \phi_1 \Rightarrow \psi \]
\[ \cdots \]
\[ \phi_n \Rightarrow \psi \]

\[ \psi \]

Negation Introduction

\[ \phi \Rightarrow \psi \]
\[ \phi \Rightarrow \neg \psi \]

\[ \neg \phi \]

Negation Elimination

\[ \neg \neg \phi \]

\[ \phi \]

Implication Introduction

\[ \phi \Rightarrow \psi \]

\[ \phi \Rightarrow \psi \]

\[ \phi \]

Implication Elimination

\[ \phi \Rightarrow \psi \]

\[ \phi \]

\[ \psi \]

Biconditional Introduction

\[ \phi \Rightarrow \psi \]
\[ \psi \Rightarrow \phi \]

\[ \phi \leftrightarrow \psi \]

Biconditional Elimination

\[ \phi \leftrightarrow \psi \]

\[ \phi \Rightarrow \psi \]
\[ \psi \Rightarrow \phi \]