Please read all instructions (including these) carefully.

The exam is closed book, closed notes, and closed internet.

The exam consists of 7 pages including this page. The last page is a reference sheet for the Fitch rules of inference. There are 5 questions. Each question is worth 10 points.

Time limit: one hour. Budget your time accordingly.

Please write your solutions in the spaces provided on the exam. Make sure that your solutions are neat and clearly marked. You may use the blank areas and backs of the exam pages for scratch work.

In accordance with both the letter and spirit of the Honor Code, I have neither given nor received assistance on this examination.

NAME: ________________________________

SUNETID (username): ________________________________

SIGNATURE: ________________________________

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Question 1 [10 points]

Please circle exactly one of the provided answers for each question, and provide a brief justification (no more than 1 sentence).

a. Is the following sentence valid, contingent, or unsatisfiable? (4 points)

\[(p \Rightarrow q) \Rightarrow r) \land ((\neg r \Rightarrow q) \Rightarrow p)\]

VALID CONTINGENT UNSATISFIABLE

b. Suppose that you are given the following premises:

\[q\]
\[p \land q \Rightarrow r\]
\[r \lor \neg p\]

i. The sentence \(r \Rightarrow p\) is consistent with the premises. (3 points)

TRUE FALSE

ii. The sentence \(r \Rightarrow p\) is logically entailed by the premises. (3 points)

TRUE FALSE
Question 2 [10 points]

Please circle exactly one of true or false for each of the following statements. No explanation is required.

1. In propositional logic, for a set of sentences $\Delta$ and a sentence $\phi$, if $\Delta \models \phi$ then $\Delta \cup \{\neg \phi\}$ is unsatisfiable.

   TRUE  FALSE

2. In propositional logic, a sentence $\phi$ logically entails a sentence $\psi$ if and only if $\phi \iff \psi$ is valid.

   TRUE  FALSE

3. In propositional resolution, the objective of the resolution proof is to arrive at the clausal form of the goal.

   TRUE  FALSE
Question 3 [10 points]

Assume that Γ, Δ are sets of sentences in Propositional Logic, and φ, ψ, and υ are sentences in Propositional Logic. Please circle true or false for each of the following statements. No justification is required.

a. If ψ ⊨ φ and φ ⊭ υ, then ψ ⊭ υ.

TRUE FALSE

b. If Γ ⊨ φ and Δ ⊨ φ, then Γ ∪ Δ ⊨ φ.

TRUE FALSE

c. If Γ ⊨ φ and Δ ⊭ φ, then Γ ∩ Δ ⊨ φ.

TRUE FALSE

d. If Γ ⊨ φ and Δ ⊨ ¬φ, then Γ ∪ Δ is unsatisfiable.

TRUE FALSE

e. If ψ ⊭ φ and ψ ⊭ ¬φ, then ψ is consistent with φ.

TRUE FALSE
Question 4 [10 points]

Given \((p \land q) \lor (\neg p \land \neg q)\) as a premise, use the Fitch system to prove \(p \Rightarrow q\).

Please annotate your proof by writing the rule of inference and line number(s) for each step in your proof (abbreviations are fine). Clearly mark any assumptions and subproofs within your proof in the same format as the exercises and notes.
Question 5 [10 points]

Part 1 [5 points]

Convert the following sentence into clausal form:

\[ \neg((p \Rightarrow b) \Rightarrow ((q \lor p) \land (z \lor x))) \]
Part 2 [5 points]

Solve the following using resolution.

Premises:
\[ a \Rightarrow q \]
\[ b \Rightarrow q \]
\[ c \Rightarrow q \]
\[ a \lor b \lor (c \land d) \]

goal:
\[ q \lor r \]
Fitch Rules of Inference

**And Introduction**
\[ \phi_1 \]
\[ \ldots \]
\[ \phi_n \]
\[ \phi_1 \land \cdots \land \phi_n \]

**And Elimination**
\[ \phi_1 \land \cdots \land \phi_n \]
\[ \phi_i \]

**Or Introduction**
\[ \phi_1 \]
\[ \phi_1 \lor \cdots \lor \phi_n \]

**Or Elimination**
\[ \phi_1 \lor \cdots \lor \phi_n \]
\[ \phi_1 \Rightarrow \psi \]
\[ \ldots \]
\[ \phi_n \Rightarrow \psi \]
\[ \psi \]

**Negation Introduction**
\[ \phi \Rightarrow \psi \]
\[ \phi \Rightarrow \neg \psi \]
\[ \neg \phi \]

**Negation Elimination**
\[ \neg \neg \phi \]
\[ \phi \]

**Implication Introduction**
\[ \phi \vdash \psi \]
\[ \phi \Rightarrow \psi \]

**Implication Elimination**
\[ \phi \Rightarrow \psi \]
\[ \phi \]
\[ \psi \]

**Biconditional Introduction**
\[ \phi \Rightarrow \psi \]
\[ \psi \Rightarrow \phi \]
\[ \phi \leftrightarrow \psi \]

**Biconditional Elimination**
\[ \phi \leftrightarrow \psi \]
\[ \phi \Rightarrow \psi \]
\[ \psi \Rightarrow \phi \]

In addition to these rules of inference, you may make **assumptions** within subproofs and use **reiteration** to reproduce an earlier conclusion in your proof.