CS 157: Midterm Examination
Fall 2017-18

- Please read all instructions (including these) carefully.
- The exam is closed book, closed notes, and closed internet.
- The exam consists of 9 pages including this page. The last page is a reference sheets for the Fitch rules of inference. There are 5 questions. Each question is worth 10 points.
- Time limit: one hour. Budget your time accordingly.
- Please write your solutions in the spaces provided on the exam. Make sure that your solutions are neat and clearly marked. You may use the blank areas and backs of the exam pages for scratch work.

In accordance with both the letter and spirit of the Honor Code, I have neither given nor received assistance on this examination.

NAME: ______________________________

SUNETID (username): ______________________________

SIGNATURE: ______________________________

<table>
<thead>
<tr>
<th>Question 1</th>
<th>Question 2</th>
<th>Question 3</th>
<th>Question 4</th>
<th>Question 5</th>
<th>Total Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>(10 points)</td>
<td>(10 points)</td>
<td>(10 points)</td>
<td>(10 points)</td>
<td>(10 points)</td>
<td>(50 points)</td>
</tr>
</tbody>
</table>
Question 1 [10 points]

Please select exactly one of the provided answers for each question.

a. Is the following sentence valid, contingent, or unsatisfiable? (4 points)

$$(\neg p \lor q) \Leftrightarrow (p \Rightarrow (q \Rightarrow p))$$

VALID  CONTINGENT  UNSATISFIABLE

b. Suppose that you are given the following premises:

$$\neg q \Rightarrow r$$
$$p \Rightarrow r$$
$$p \lor \neg q$$

Is $r$ entailed by the premises? (3 points)

YES  NO

Is $q$ consistent with the premises? (3 points)

YES  NO
Question 2 [10 points]

Please circle exactly one of true or false for each of the following statements. If a statement is false, provide an appropriate counter example.

a. In Propositional Logic, the Fitch system is sound. (2.5 points)
   
   TRUE  FALSE

b. In Propositional Logic, the Fitch system is complete. (2.5 points)
   
   TRUE  FALSE

c. Propositional Resolution is sound. (2.5 points)
   
   TRUE  FALSE

d. Propositional Resolution is complete. (2.5 points)
   
   TRUE  FALSE
Question 3 [10 points]

Assume that $\Gamma$ and $\Delta$ are sets of sentences in Propositional Logic, and $\phi$ and $\psi$ are sentences in Propositional Logic. Select exactly one of true or false for each of the following statements. No justification is required.

a. If $\Delta \models \phi$, then $\Delta \cup \{\neg \phi\}$ is unsatisfiable. (2 points)

   TRUE    FALSE

b. If $\{\phi\} \models \psi$ and $\{\psi\} \models \phi$, then $\{\} \models (\phi \iff \psi)$. (2 points)

   TRUE    FALSE

c. $\Delta \subseteq \Gamma$ and $\Gamma \models \phi$, then $\Delta \models \phi$. (2 points)

   TRUE    FALSE

d. $\Gamma \models \phi$ and $\Delta \models \phi$, then $\Gamma \cup \Delta \models \phi$. (2 points)

   TRUE    FALSE

ee. If $\Gamma \models (\phi \lor \psi)$, then $\Gamma \models \phi$ or $\Gamma \models \psi$. (2 points)

   TRUE    FALSE
Question 4 [10 points]

a. Given the premises

\[ \neg q \lor \neg r \]
\[ r \]

use the Fitch system to prove \( \neg q \). Please annotate your proof by writing the rule and line number for each step in your proof (abbreviations are fine). Clearly mark any assumptions and subproofs within your proof in the same format as the exercises and notes. (5 points)
b. Given the premises

\[ p \rightarrow q \]
\[ \neg q, \]

use the Fitch system to prove \( \neg p \). Please annotate your proof by writing the rule and line number for each step in your proof (abbreviations are fine). Clearly mark any assumptions and subproofs within your proof in the same format as the exercises and notes. (5 points)
Question 5 [10 points]

a. Convert the following sentence to clausal form. (7 points)

\[-((p \Rightarrow q \land w) \Rightarrow (\neg w \Rightarrow \neg p \land \neg r))\]

*Hint:* Our conversion yields 4 clauses.
b. Use resolution to show that the following set of clauses is unsatisfiable. Please annotate your proof by writing the rule and line number for each step in your proof (abbreviations are fine). (3 points)

1. \{p, \neg q, \neg r\}
2. \{q, \neg r\}
3. \{r\}
4. \{\neg p\}
Fitch Rules of Inference

And Introduction
φ₁
...
φₙ

__________
φ₁ ∧ ··· ∧ φₙ

And Elimination
φ₁ ∧ ··· ∧ φₙ

__________
φᵢ

Or Introduction
φ₁

__________
φ₁ ∨ ··· ∨ φₙ

Or Elimination
φ₁ ∨ ··· ∨ φₙ
φ₁ ⇒ ψ
...
φₙ ⇒ ψ

__________
ψ

Negation Introduction
φ ⇒ ψ
φ ⇒ ¬ψ

__________
¬φ

Negation Elimination
¬¬φ

__________
φ

Implication Introduction
φ ⊨ ψ

__________
φ ⇒ ψ

Implication Elimination
φ ⇒ ψ
φ

__________
ψ

Biconditional Introduction
φ ⇒ ψ
ψ ⇒ φ

__________
φ ⇔ ψ

Biconditional Elimination
φ ⇔ ψ

__________
φ ⇒ ψ
ψ ⇒ φ

In addition to these rules of inference, you may make assumptions within subproofs and use reiteration to reproduce an earlier conclusion in your proof.