CS 157 Midterm Examination
Fall 2016-17

- Please read all instructions (including these) carefully.
- The exam is closed book, closed notes, and closed internet.
- The exam consists of 8 pages including this page. The last page is a reference sheets for the Fitch rules of inference. There are 5 questions. Each question is worth 10 points.
- Time limit: one hour. Budget your time accordingly.
- Please write your solutions in the spaces provided on the exam. Make sure that your solutions are neat and clearly marked. You may use the blank areas and backs of the exam pages for scratch work.

In accordance with both the letter and spirit of the Honor Code, I have neither given nor received assistance on this examination.

NAME:  

SUNETID (username):  

SIGNATURE:  

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<tr>
<th>Question 1 (10 points)</th>
<th>Question 2 (10 points)</th>
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Question 1 [10 points]

(a) Suppose that you are given the following premises:

\[ p \land q \Rightarrow r \]
\[ q \lor \neg p \]
\[ p \]

Please circle one of the provided answers for each question, and provide a brief justification (no more than 1 sentence).

1) The sentence \( r \) is **consistent** with the supplied premises. (3 points)

   TRUE  \hspace{1cm} FALSE

2) The sentence \( r \) is **logically entailed** by the supplied premises. (3 points)

   TRUE  \hspace{1cm} FALSE

(b) Is the following sentence **valid**, **contingent**, or **unsatisfiable**? (4 points)

\[ (p \Rightarrow q) \lor (\neg q \land \neg p \Rightarrow r) \]

VALID  \hspace{1cm} CONTINGENT  \hspace{1cm} UNSATISFIABLE
Question 2 [6 points]

(a) The Fitch system for Propositional Logic is sound. (1 point)

TRUE       FALSE

(b) The Fitch system for Propositional Logic is complete. (2 points)

TRUE       FALSE

(c) Propositional Resolution is sound. (1 point)

TRUE       FALSE

(d) Propositional Resolution is (not necessarily generatively) complete. (2 points)

TRUE       FALSE
**Question 3 [9 points]**

Assume that \( \Gamma \) and \( \Delta \) are sets of sentences in Propositional Logic, and \( \phi \) and \( \psi \) are individual sentences in Propositional Logic. Please circle *true* or *false* for each of the following statements. No explanation is required.

(a) If \( \Gamma \vDash \neg \phi \) and \( \Delta \vDash \phi \), then \( \Gamma \cup \Delta \vDash \phi \). (3 points)

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<th>TRUE</th>
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(b) If \( \Gamma \vDash \neg \phi \) and \( \Delta \vDash \phi \), then \( \Gamma \cap \Delta \vDash \phi \). (3 points)

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(c) If \( \Gamma \cup \Delta \vDash \phi \) and \( \Delta \vDash \phi \), then \( \Gamma \vDash \phi \) or \( \Delta \vDash \phi \). (3 points)

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Question 4 [15 points]

Use the Fitch System to prove \((\neg r \Rightarrow \neg p) \Rightarrow (q \land r)\) from the premises \((p \Rightarrow r) \Rightarrow (r \land \neg p)\) and \(\neg p \Rightarrow q\).

Please annotate your proof by writing the rule and line numbers for each step in your proof (abbreviations are fine). Clearly mark any assumptions and subproofs within your proof in the same format as the exercises and notes.

\[
\begin{align*}
(1) \ (p \Rightarrow r) & \Rightarrow (r \land \neg p) \\
(2) & \ (\neg p \Rightarrow q)
\end{align*}
\]
Question 5 [10 points]

(a) Convert the following two logical sentences to clausal form. (5 points)

\[ \neg((p \lor \neg q) \Rightarrow (p \land q \land \neg r)) \]
\[ \neg(p \lor q) \Rightarrow \neg r \]
(b) Prove \( \neg q \) with Propositional Resolution using the sentences from (a) as premises. (5 points)
Fitch Rules of Inference and Induction Axioms

And Introduction
\[
\begin{align*}
\phi_1 \\
\vdots \\
\phi_n \\
\hline \\
\phi_1 \land \ldots \land \phi_n
\end{align*}
\]

And Elimination
\[
\begin{align*}
\phi_1 \land \ldots \land \phi_n \\
\hline \\
\phi_i
\end{align*}
\]

Or Introduction
\[
\begin{align*}
\phi_i \\
\hline \\
\phi_1 \lor \ldots \lor \phi_n
\end{align*}
\]

Or Elimination
\[
\begin{align*}
\phi_1 \lor \ldots \lor \phi_n \\
\phi_1 \Rightarrow \psi \\
\vdots \\
\phi_n \Rightarrow \psi \\
\hline \\
\psi
\end{align*}
\]

Negation Introduction
\[
\begin{align*}
\phi \Rightarrow \psi \\
\phi \Rightarrow \neg \psi \\
\hline \\
\neg \phi
\end{align*}
\]

Negation Elimination
\[
\neg \neg \phi \\
\hline \\
\phi
\]

Implication Introduction
\[
\begin{align*}
\phi \vdash \psi \\
\hline \\
\phi \Rightarrow \psi
\end{align*}
\]

Implication Elimination
\[
\begin{align*}
\phi \Rightarrow \psi \\
\phi \\
\hline \\
\psi
\end{align*}
\]

Biconditional Introduction
\[
\begin{align*}
\phi \Rightarrow \psi \\
\psi \Rightarrow \phi \\
\hline \\
\phi \equiv \psi
\end{align*}
\]

Biconditional Elimination
\[
\begin{align*}
\phi \equiv \psi \\
\hline \\
\phi \Rightarrow \psi \\
\psi \Rightarrow \phi
\end{align*}
\]